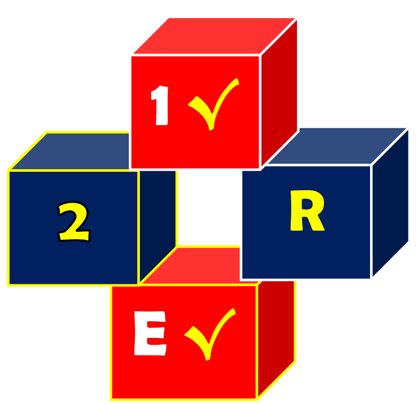
Simplest Maths Booklet

Explaning and Exercises

Algebra and Geometry

Prep 3 - First Term 2024



Prepared by :
Mr.Mohamed El-Shourbagy

Simplest Maths

Maths-Prep3/first term (2024)





1. Cartesian product.
2. Relation - Function (mapping).
3. The symbolic representation of the function - Polynomial functions.
4. The study of some polynomial functions.



1. Ratio and proportion.
2. Follow properties of proportion.
3. Continued proportion.
4. Direct variation and inverse variation.



1. Collecting data.
2. Dispersion.

Mr.Mohamed El-Shourbagy / 01093149109



Lesson (1)

Cartesian Product

Unit 1

The ordered pair

(a , b) is called an ordered pair

- a is called the first projection
- b is called the second projection

Notice that :

1. $(a , b) \neq \{a , b\}$, $(a , b) \neq [a , b]$
2. The element in the ordered pair can be repeated while that cannot happen in the sets.

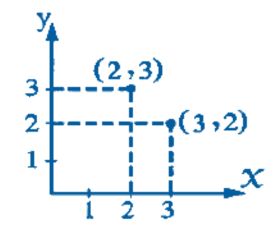
For example :

We say the ordered pair (2 , 2) while we cannot say $\{2 , 2\}$ but we say $\{2\}$

3. There is an empty set which is denoted by $\varnothing$ while there is not an empty ordered pair.
4. $(a , b) \neq (b , a)$ where $a \neq b$

For example : $(2 , 3) \neq (3 , 2)$

Notice that : (2 , 3) and (3 , 2) are represented by two different points as shown in the opposite graph.



The equality of two ordered pairs

If $(a , b) = (x , y)$, then $a = x$, $b = y$

For example :

- If $(a , b) = (3 , -4)$, then $a = 3$, $b = -4$
- If $(x , 2) = (-5 , y)$, then $x = -5$, $y = 2$

Example

Find the values of x and y in each of the following if :

1. $(x^2 - 1 , 8) = (48 , \sqrt[3]{y})$
2. $(32 , x + y) = (y^5 , 2)$

Solution

1. $\because (x^2 - 1 , 8) = (48 , \sqrt[3]{y})$

$\therefore x^2 - 1 = 48$ $\qquad \therefore x^2 = 49$ $\qquad \therefore x = \pm 7$,

$\sqrt[3]{y} = 8$ $\qquad \therefore y = 8^3$ $\qquad \therefore y = 512$

2 $\because (32 , x + y) = (y^5 , 2)$ $\therefore y^5 = 32$ $\therefore y = 2$ «for $2^5 = 32$»

, $x + y = 2$ and by substituting by the value of y :

$\therefore x + 2 = 2$ $\therefore x = 0$

The Cartesian product of two finite sets and representing it

For any two finite and non empty sets X and Y, we get :

1 The Cartesian product of the set X by the set Y and it is denoted by $Y \times X$ is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y **i.e.** $X \times Y = \{(a , b) : a \in X , b \in Y\}$

For example :

If $X = \{1 , 2\}$, $Y = \{5 , 7 , 8\}$, then

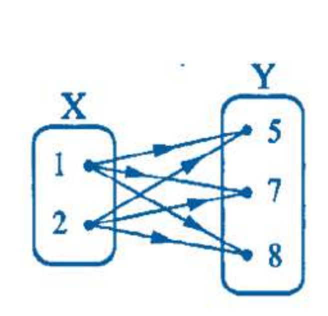
$X \times Y = \{1 , 2\} \times \{5 , 7 , 8\}$

$= \{(1 , 5) , (1 , 7) , (1 , 8) , (2 , 5) , (2 , 7) , (2 , 8)\}$

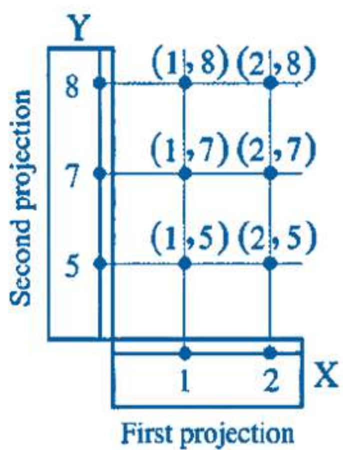
The opposite table helps us to get $X \times Y$

	×	Second projection 5	7	8
First projection	1	(1 , 5)	(1 , 7)	(1 , 8)
	2	(2 , 5)	(2 , 7)	(2 , 8)

We can represent $X \times Y$ by an arrow diagram or graphical (Cartesian) diagram as follows :



The arrow diagram



The graphical diagram (The Cartesian diagram)

2 The Cartesian product of the set Y by the set X and which is denoted by $Y \times X$ is the set of all ordered pairs whose first projection belongs to the set Y and the second projection belongs to the set X

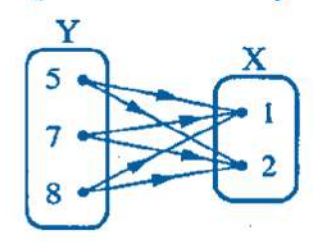
i.e. $Y \times X = \{(a , b) : a \in Y , b \in X\}$

↘ For example :

If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then $Y \times X = \{5, 7, 8\} \times \{1, 2\}$

$= \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$

We can represent $Y \times X$ by an arrow diagram or by a Cartesian diagram as follows :



The arrow diagram

X
Second projection
2
1
(5, 2) (7, 2) (8, 2)
(5, 1) (7, 1) (8, 1)
5 7 8
Y
First projection

The Cartesian diagram

Remarks

From the previous, we notice that :

- **$X \times Y \neq Y \times X$ where $X \neq Y$**

  because $(1, 5) \neq (5, 1)$

We say $X \times Y = Y \times X$ at the following cases :

(1) $X = Y$

(2) One of the two sets $= \varnothing$

« $X \times \varnothing = \varnothing \times X = \varnothing$ because $\varnothing$ has no elements »

3 The Cartesian product of the set X by itself and we denote it by $X \times X$ in the same times it is denoted by X^2 (it is read X two) is the set of all ordered pairs whose first projections and second projections belong to X

i.e. $X \times X = \{(a, b) : a \in X, b \in X\}$

↘ For example :

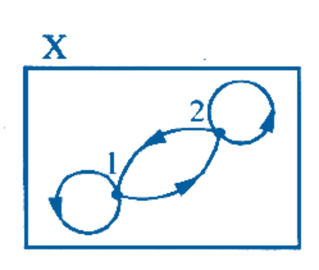
If $X = \{1, 2\}$, then

$X \times X = \{1, 2\} \times \{1, 2\}$

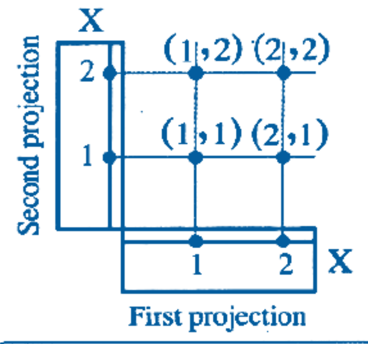
$= \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

	⊗	**Second projection**	
		1	**2**
First projection	**1**	(1, 1)	(1, 2)
	2	(2, 1)	(2, 2)

We can represent $X \times X$ by an arrow diagram or Cartesian diagram as follows :



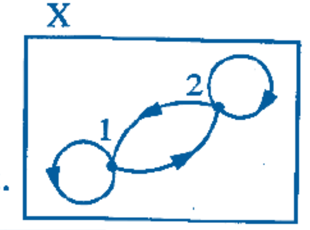
The arrow diagram



Cartesian diagram

Remark

The ordered pairs in which the first projection equals the second projection in the previous Cartesian product (1 , 1) , (2 , 2) are represented in the arrow diagram by a loop ◯ to show that the arrow goes and returns to the same point.



Example

If $X = \{2 , 3 , 4\}$ and $Y = \{a , b\}$ find each of :

1 $X \times Y$ **2** $Y \times X$ **3** $X \times X$

, then find the number of elements of each of them.

Solution

1 $X \times Y = \{(2 , a) , (2 , b) , (3 , a) , (3 , b) , (4 , a) , (4 , b)\}$, the number of elements of $X \times Y = 6$ ordered pairs.

2 $Y \times X = \{(a , 2) , (a , 3) , (a , 4) , (b , 2) , (b , 3) , (b , 4)\}$, the number of elements of $Y \times X = 6$ ordered pairs.

3 $X \times X = \{(2 , 2) , (2 , 3) , (2 , 4) , (3 , 2) , (3 , 3) , (3 , 4) , (4 , 2) , (4 , 3) , (4 , 4)\}$, the number of elements of $X \times X = 9$ ordered pairs.

Remarks

If we denote the number of elements of any set by «n» then from the previous example , we find that :

- $n(X) = 3$, $n(Y) = 2$

i.e. **1** $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$

2 $n(X \times X) = (n(X))^2$

Remark

If : $(a, b) \in X \times Y$, then $a \in X$, $b \in Y$

For example :

If : $(3, 5) \in X \times Y$, then $3 \in X$, $5 \in Y$

Example

If : $X = \{1, 2\}$, $Y = \{3, 7\}$ and $Z = \{3\}$ **Find :**

(1) $X \times Z$ (2) $n(Y^2)$ (3) $(Y \cap Z) \times X$

Solution

(1) $X \times Z = \{(1, 3), (2, 3)\}$

(2) $n(Y^2) = 2 \times 2 = 4$

(3) $(Y \cap Z) \times X = \{(3, 1), (3, 2)\}$

Example

If : $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :**

(1) $X \times (Y \cap Z)$ (2) $(X - Y) \times Z$

Solution

(1) $X \times (Y \cap Z) = \{(3, 5), (4, 5)\}$ (2) $(X - Y) \times Z = \{(3, 6), (3, 5)\}$

Example

If : $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :**

(1) $X \times (Y \cap Z)$ (2) $(X - Y) \times Z$

(3) $(X - Y) \times (Y - Z)$

Solution

(1) $X \times (Y \cap Z) = \{(3, 5), (4, 5)\}$

(2) $(X - Y) \times Z = \{(3, 6), (3, 5)\}$ (3) $(X - Y) \times (Y - Z) = \{(3, 4)\}$

The Cartesian product of two infinite sets

- We know that if X is a finite set (having n elements) , then the Cartesian product X × X is also a finite set (having n^2 elements).

For example: If n (X) = 3 , then n (X × X) = 9

- But if X is an infinite set , then X × X is an infinite set also

As examples for that

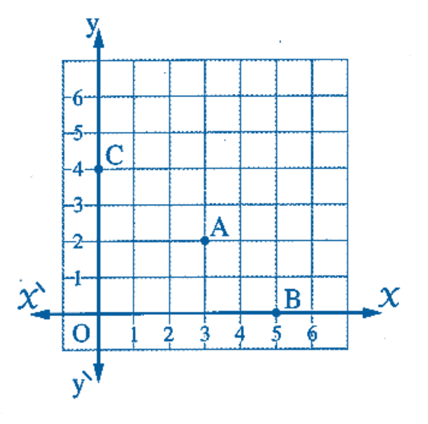
$\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,
$\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

Representing the Cartesian product of two infinite sets

- We know that if X is a finite set , we represent the Cartesian product X × X graphically by a finite number of points.
- But if X is an infinite set , then the Cartesian product X × X represented graphically by an infinite number of points.

First Representing the Cartesian product $\mathbb{N} \times \mathbb{N}$ ($\mathbb{N}^2$)

- Represent the natural numbers on two perpendicular straight lines, one of them $\overleftrightarrow{xx'}$ is horizontal and the other $\overleftrightarrow{yy'}$ is vertical , where they intersect at the point which represents the number zero on each of them *i.e.* O = (0 , 0)
- And the opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ which consists of the vertical and the horizontal straight lines that pass through the points which represent the natural numbers on each of $\overleftrightarrow{xx'}$ and $\overleftrightarrow{yy'}$



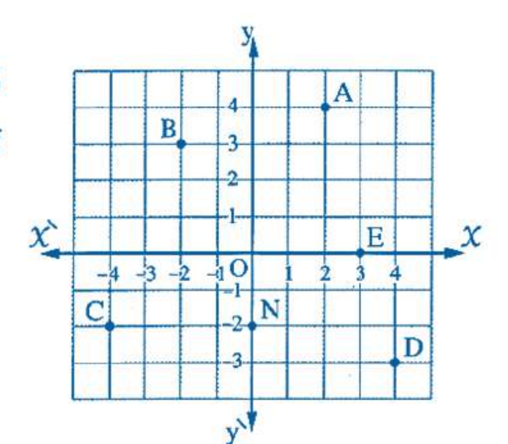
- And each point of the points of this net represents an ordered pair of the Cartesian product $\mathbb{N} \times \mathbb{N}$

For example:

- The point A represents the ordered pair (3 , 2)
- The point B represents the ordered pair (5 , 0)
- The point C represents the ordered pair (0 , 4)
- The point O represents the ordered pair (0 , 0)

Second Representing the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ ($\mathbb{Z}^2$)

- Represent the integers on each of $\overleftrightarrow{xx'}$ and $\overleftrightarrow{yy'}$ which are intersecting at O (0 , 0)
- And the opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$
- And each point of its points represents an ordered pair of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$

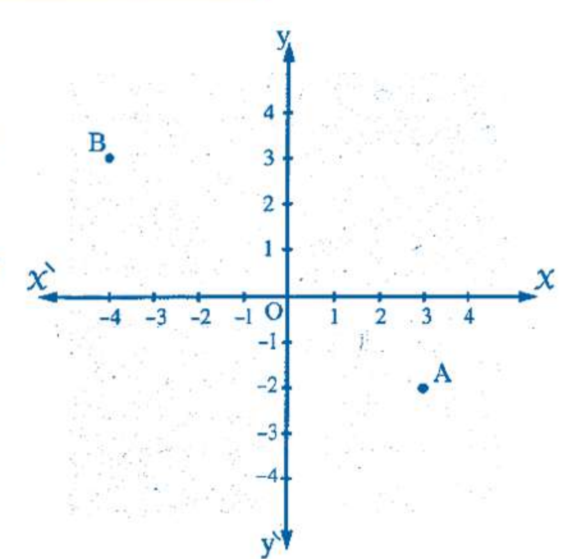


For example:

- The point A represents the ordered pair (2 , 4)
- The point B represents the ordered pair (– 2 , 3)
- The point C represents the ordered pair (– 4 , – 2)
- The point D represents the ordered pair (4 , – 3)
- The point E represents the ordered pair (3 , 0)
- The point N represents the ordered pair (0 , – 2)

Third Representing the Cartesian product $\mathbb{R} \times \mathbb{R}$ ($\mathbb{R}^2$)

- The perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$ is an infinite extended surface from all sides and the opposite figure shows a small part of this region.
- Each point of this region represents an ordered pair of the Cartesian product $\mathbb{R} \times \mathbb{R}$

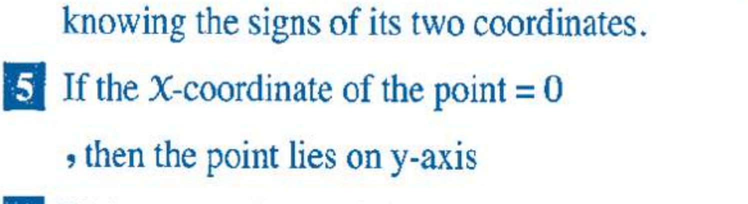


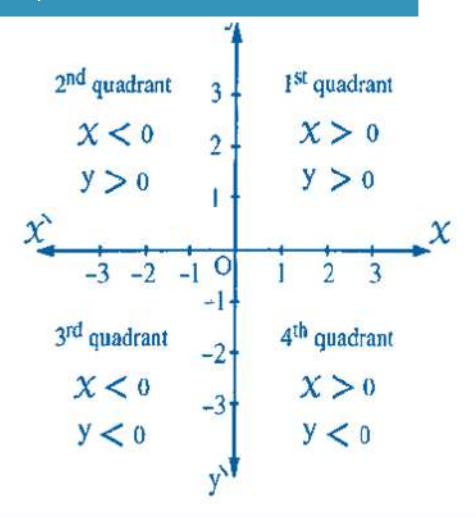
For example:

- The point A represents the ordered pair (3 , – 2)
- The point B represents the ordered pair (– 4 , 3)

Remarks

1. The horizontal straight line $\overleftrightarrow{xx'}$ is called x-axis or the horizontal axis and the vertical straight line $\overleftrightarrow{yy'}$ is called y-axis or the vertical axis.
2. The point of intersection of the two axes $\overleftrightarrow{xx'}$ and $\overleftrightarrow{yy'}$ is called the origin point.
3. If the point A represents the ordered pair (x , y) in the Cartesian product $\mathbb{R} \times \mathbb{R}$, then :
   - The first projection x is called the x-coordinate of the point A
   - The second projection y is called the y-coordinate of the point A


4 The two axes $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.




5 If the x-coordinate of the point = 0

, then the point lies on y-axis

6 If the y-coordinate of the point = 0

, then the point lies on x-axis

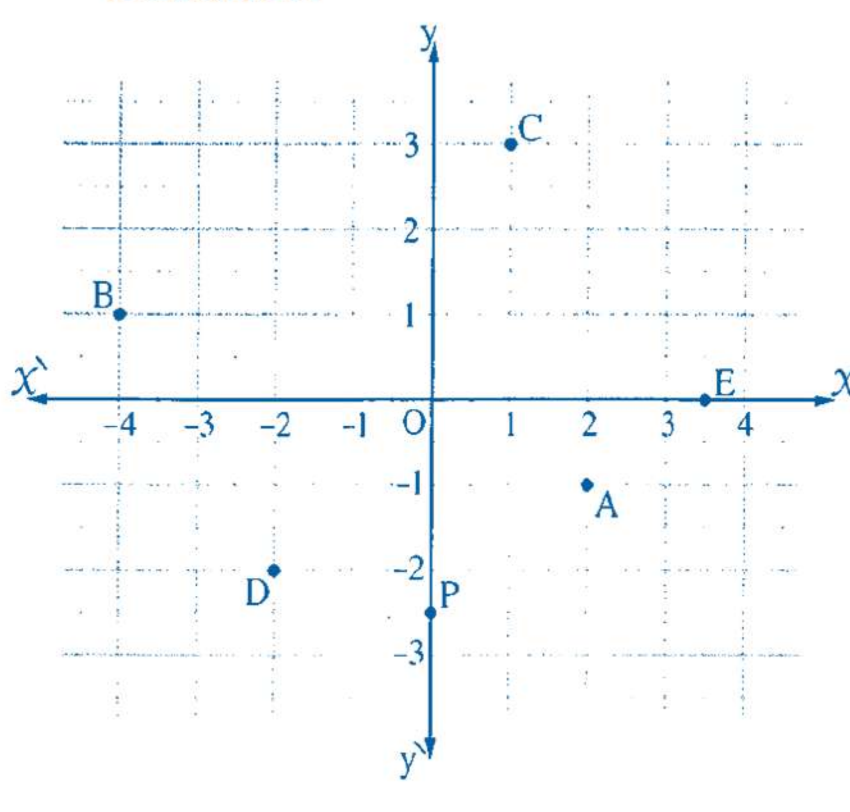
Example

Mention the quadrant or the axis on which each of the following points lies on the perpendicular square net of the Cartesian product $\mathbb{R} \times \mathbb{R}$, then locate it on the net :

A $(2 , -1)$, B $(-4 , 1)$, C $(1 , 3)$, D $(-2 , -2)$, E $\left(3\frac{1}{2} , 0\right)$, P $\left(0 , -2\frac{1}{2}\right)$

Solution

- A $(2 , -1)$ lies on the 4^{th} quadrant
- B $(-4 , 1)$ lies on the 2^{nd} quadrant
- C $(1 , 3)$ lies on the 1^{st} quadrant
- D $(-2 , -2)$ lies on the 3^{rd} quadrant
- E $\left(3\frac{1}{2} , 0\right)$ lies on the x-axis
- P $\left(0 , -2\frac{1}{2}\right)$ lies on the y-axis



EX. (1): Choose the correct answer:

Exercises

1 If : $(x^3 , y + 3) = (1 , \sqrt{4})$, then : $x - y =$ ....
(a) 3 (b) 2 (c) 1 (d) zero

2 If : $(a + 1 , 5) = (-2 , b - 1)$, then : $2a + b =$ .... *(El-Ismailia 2014)*
(a) – 12 (b) zero (c) 2 (d) 12

3 If : $X = \{3\}$, then : $X^2 =$ .... *(Cairo 2013)*
(a) 9 (b) (3 , 3) (c) {9} (d) {(3 , 3)}

4 If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ ....
(a) 1 (b) 2 (c) 5 (d) zero

5 If : $n(X) = 3$, $Y = \{4 , 5\}$, then $n(X \times Y) =$ ....
(a) 2 (b) 3 (c) 5 (d) 6

6 If : $X = \{5 , 6 , 7\}$, then $n(X^2) =$ ....
(a) 3 (b) 6 (c) 9 (d) 12

7 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ .... *(El-Kalyoubia 2011)*
(a) 4 (b) 9 (c) 15 (d) 36

8 If : $n(X) = 5$, $n(X \times Y) = 15$, then $n(Y) =$ ....
(a) 1 (b) 5 (c) 3 (d) 15

9 If : $X \times Y = \{(1 , 3) , (1 , 4)\}$, then $n(X) =$ ....
(a) 1 (b) 2 (c) 3 (d) 4

10 If : $(2, 5) \in \{2 , 6\} \times \{x , 8\}$ then $x =$ ....
(a) 8 (b) 6 (c) 5 (d) 3

11 If : $(X - Y) \times Y = \{(1, 2), (1, 3)\}$ and $n(X \times Y) = 6$, then $X =$ ....

(a) $\{1\}$ (b) $\{1, 2\}$ (c) $\{1, 3, 6\}$ (d) $\{1, 3, 2\}$

12 The point $(-5, 7)$ lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

13 The point $(-3, 4)$ lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

14 The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2, 0)$ and ....

(a) $(0, 2)$ (b) $(2, 5)$ (c) $(5, 2)$ (d) $(-2, 2)$

15 The point $A(5, -3)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

16 If the point $(5, b - 5)$ is located on the X-axis then $b =$ ....

(a) zero (b) -5 (c) 5 (d) 10

17 If the point $(5, b - 7)$ is located on the X-axis, then $b =$ .... *(Alex. 2011)*

(a) 2 (b) 5 (c) 7 (d) 12

18 If : $(|X|, 4) = (3, y^2)$ and the point (X, y) lies in the second quadrant, then $X + y =$ .... *(El-Sharkia 2014)*

(a) 7 (b) 1 (c) -1 (d) -7

19 If the point $(X - 5, 3 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant, then X equals ....

(a) 2 (b) 3 (c) 4 (d) 5

20	If : $(5 , x-8) = (y+1 , -5)$, then : $x + y =$ .... *(Aswan 2011)* (a) 4 (b) 5 (c) 6 (d) 7
21	If : $(2 , x-1) = (y , 0)$, then $x + y =$ .... (a) 3 (b) 1 (c) 2 (d) -3
22	If : $X = \{3\}$, then : $X^2 =$ .... *(Cairo 2013)* (a) 9 (b) $(3 , 3)$ (c) $\{9\}$ (d) $\{(3 , 3)\}$
23	If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ .... (a) 1 (b) 2 (c) 5 (d) zero
24	If : $n(X) = 2$, $Y = \{1 , 2\}$, then $n(X \times Y) =$ .... (a) 4 (b) 3 (c) 5 (d) 6
25	If : $n(X) = 3$, $Y = \{2 , 5\}$ then $n(X \times Y) =$ .... (a) 2 (b) 3 (c) 5 (d) 6
26	If : $X = \{3\}$ and $n(Y) = 4$, then $n(X \times Y) =$ .... (a) 1 (b) 4 (c) 7 (d) 12
27	If : $X = \{5\}$, $Y = \{3\}$, then $n(X \times Y) =$ .... (a) 15 (b) 5 (c) 3 (d) 1
28	If : $X = \{3 , 5 , 6\}$, then $n(X^2) =$ .... (a) 3 (b) 6 (c) 9 (d) 12
29	If : $X \times Y = \{(6 , 3) , (6 , 4)\}$, then $n(X) =$ .... (a) 3 (b) 1 (c) 4 (d) 2
30	🕮 If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x =$ .... *(El-Behaira 2011)* (a) 8 (b) 6 (c) 5 (d) 3

31	The point $(-5 , 3)$ lies in ………… quadrant. (a) first (b) second (c) third (d) fourth
32	The point $(-4 , 3)$ lies in the ………… quadrant. (a) first (b) second (c) third (d) fourth
33	The point $(-2 , -5)$ lies in the ………… quadrant. (a) first (b) second (c) third (d) fourth
34	If : $(\|X\| , 4) = (3 , y^2)$ and the point (X , y) lies in the second quadrant , then : $X + y =$ ………… (a) 7 (b) 1 (c) -1 (d) -7
35	The cartesian product $\{2\} \times \mathbb{R}$ represent graphically by a straight line passing through the two points $(2 , 0)$ and ………… (a) $(0 , 2)$ (b) $(2 , 5)$ (c) $(5 , 2)$ (d) $(-2 , 2)$
36	The point $(5 , -2)$ lies on the ………… quadrant. (a) first (b) second (c) third (d) fourth
37	If the point $(5 - X , X - 4)$ lies in the fourth quadrant , then the value of $X =$ ………… (a) 9 (b) 8 (c) 6 (d) 2
38	If the point $(X , 2)$ lies on y-axis , then $X =$ ……… *(El-Fayoum 2011)* (a) zero (b) 1 (c) 2 (d) 3
39	If the point $(X , 7)$ lies on y-axis , then $X =$ ……… *(New vally 2013)* (a) zero (b) 7 (c) -7 (d) 49
40	If : $X = \{5\}$, $Y = \varnothing$, then $n(X \times Y) =$ ………… (a) 1 (b) 2 (c) 5 (d) zero

EX. (2): Answer the following:

1	**Find :** a , b if $(a-7, 26) = (-2, b^3-1)$
2	If $(x-1, 11) = (8, y+3)$, **then find :** $\sqrt{x+2y}$
3	If $(x^2, 27) = (1, y^3)$ and the point (x, y) lies in the second quadrant , **find the value of :** $\sqrt{y-x}$
4	If $(2x, 4) = (8, y+1)$, **then find the value of :** $\sqrt{x^2+y^2}$
5	📖 **If** $X = \{2, -1\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$, **find :** [1] $X \times Y$ [2] $Y \times Z$ [3] X^2 [4] $n(X \times Z)$ [5] $n(Y^2)$ [6] $n(Z^2)$
6	📖 **If** $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$, **find :** [1] X and Y [2] $Y \times X$ [3] Y^2
7	**If** $X = \{3, 4, 8\}$ **Find** X^2 **and represent it :** [1] By an arrow diagram. [2] By Cartesian diagram.
8	📖 **If** $X = \{3, 4\}$, $Y = \{4, 5\}$ **and** $Z = \{6, 5\}$, **then find :** [1] $X \times (Y \cap Z)$ [2] $(X - Y) \times Z$ [3] $(X - Y) \times (Y - Z)$
9	📖 **If** $X = \{1, 2\}$, $Y = \{3, 4, 5\}$ **Find** $X \times Y$ **and represent it by :** [1] The arrow diagram. [2] The Cartesian diagram.
10	If $X \times Y = \{(2, 3), (2, 2), (2, 4)\}$ **Find each of the following :** (1) X , Y (2) $X \times (X \cap Y)$

11	📖 If $X = [-2, 3]$, find the location which represents $X \times X$ Show which of the following points belongs to the Cartesian product of $X \times X$ $A(1, 2)$, $B(3, -1)$, $C(-1, 4)$ and $D(-2, 0)$
12	If $X = \{2, 15\}$, $Y = \{4, 1\}$ and $Z = \{15\}$ **Find :** (1) $Y \times Z$ (2) $n(X^2)$ (3) $(X \cap Z) \times Y$
13	📖 **Identify the following points on a perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$:** $A(4, 5)$, $B(6, -3)$, $C(-2, 7)$, $D(-1, 6)$, $E(-4, -5)$, $M(0, 6)$, $K(9, 0)$ Then mention the quadrant that each point is located on the perpendicular graphical net or the axis it belongs to.
14	📖 **If $X = \{1\}$, $Y = \{2, 3\}$, $Z = \{2, 5, 6\}$** **Represent each of X , Y and Z by Venn diagram , then find :** **First :** [1] $X \times Y$ [2] $Y \times Z$ [3] $X \times Z$ [4] Y^2 **Second :** $(X \times Y) \cup (Y \times Z)$ **Third :** $X \times (Y \cap Z)$ **Fourth :** $(X \times Y) \cap (X \times Z)$ **Fifth :** $(Z - Y) \times (X \cup Y)$

Explaining

Lesson (2)

Relation – function (mapping)

First The relation

Remarks

1. The relation R is a subset of the Cartesian product $X \times Y$ *i.e.* $R \subset X \times Y$
2. If $(a, b) \in$ the relation R, then we can express that by another method, we write "a R b", it means that the element a is connected with the element b by the relation R

The conclusion

1. The relation from a set X to a set Y is a connection joining some or all the elements of X with some or all the elements of Y
2. If R is a relation from the set X to the set Y, then R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y and the first projection connects with the second projection with respect to this relation.
3. The relation R from the set X to the set Y is a subset from the Cartesian product $X \times Y$
   i.e. The relation $R \subset X \times Y$
   Inversely : any subset of the Cartesian product $X \times Y$ expresses a relation from X to Y
4. The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically).

Remark

If R is a relation from X to X, then : R is a relation on X and the relation $R \subset X \times X$

Second Functions (Mapping)

Generally

A relation from X to Y is said to be a function if :

1. In the relation, each element of the set X appears only once as a first projection in one of the ordered pairs of the relation. (Notice the relation R in the previous example)
2. In the arrow diagram which represents the relation, each element of X has one and only one arrow going out of it to one element of Y
   (Notice the arrow diagram of the previous relation)
3. In the Cartesian diagram which represents the relation, each vertical line has one and only one point lying on it of the points which represent the relation.
   (Notice the Cartesian diagram of the previous relation)

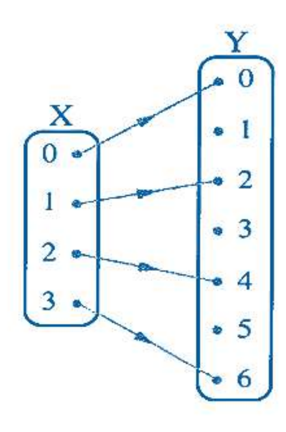
Example

If $X = \{0, 1, 2, 3\}$, $Y = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2} b$" for each $a \in X$, $b \in Y$

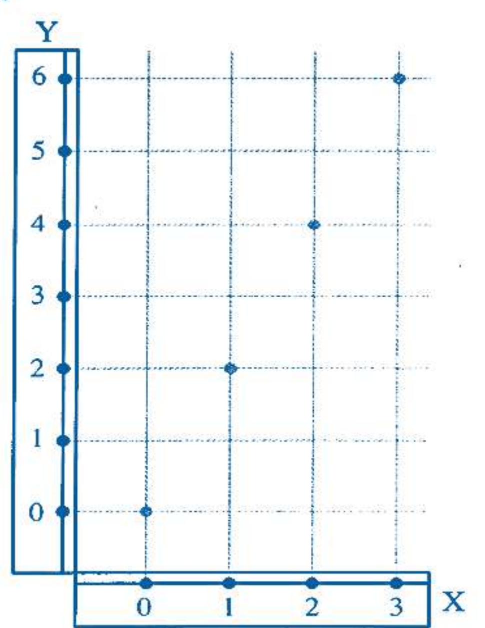
Write R and represent it by an arrow diagram and a Cartesian diagram.

Solution

$R = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$



The arrow diagram



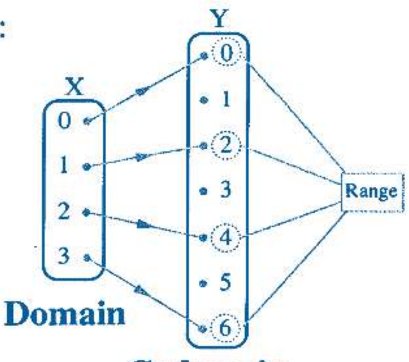
The Cartesian diagram

In the previous relation, we notice that :

Each element of the set X has been connected with **one and only one element** of the elements of the set Y

Such as, this relation is called a function or (mapping), also :

- The set of $X = \{0, 1, 2, 3\}$ is called "the domain of the function".
- The set of $Y = \{0, 1, 2, 3, 4, 5, 6\}$ is called "the codomain of the function".
- The set $\{0, 2, 4, 6\}$ is called "the range of the function" and it is a subset from the codomain of the function.



Prime numbers = { 2 , 3 , 5 , 7 , 11 , 13 , 17 , 19 , 23 , 29 , 31 , 37 }

Odd numbers = { 1 , 3 , 5 , 7 , 9 , 11 , 13 , 15 , 17 , 19 , ... }

Even numbers = { 0 , 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , 20 , ... }

Example

If $X = \{0, 1, 2, 3\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y

where "a R b" means "a + b = 5" for each $a \in X$, $b \in Y$

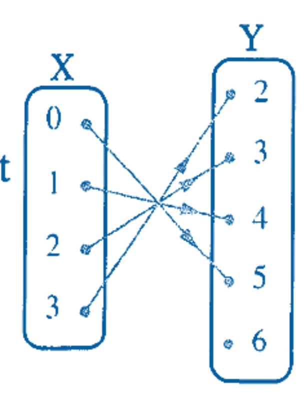
Write the relation R and represent it by an arrow diagram.

Mention giving reasons if R is a function from X to Y or not ?

And if it is a function , find its range.

Solution

- $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$



R represents a function from X to Y because each element of X connects with only one element of Y

The range of the function = $\{5, 4, 3, 2\}$

Example

If $X = \{3, 2, 1, 0, \frac{1}{2}, \frac{1}{3}\}$, and R is a relation on X

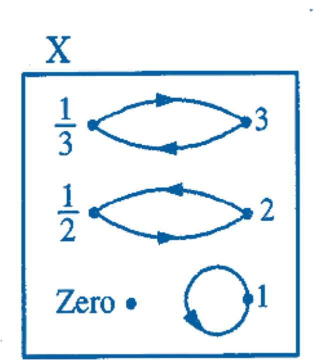
where "a R b" means "a is the multiplicative inverse of b"

for each $a \in X$, $b \in X$

Write R and represent it by an arrow diagram and mention giving reasons if R represents a function or not.

Solution

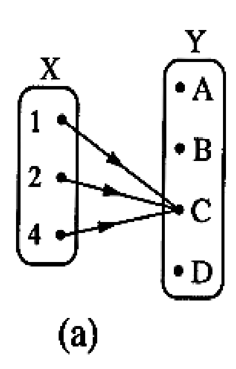
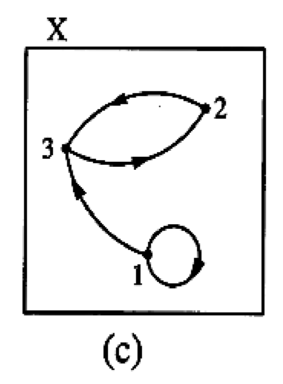
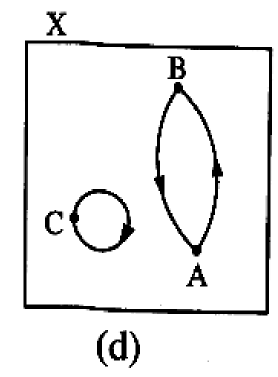
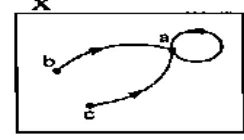
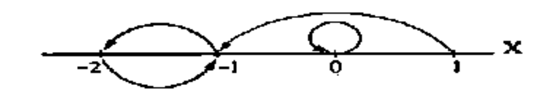
- $R = \{(3, \frac{1}{3}), (2, \frac{1}{2}), (1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3)\}$

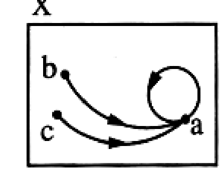


R does not represent a function because the element zero $\in X$ does not connect with any element in X (There is no arrow going out from zero in the arrow diagram which represents the relation)

EX. (1): Choose the correct answer:

Exercises

1	If f is a function from the set X to the set Y then the domain of f is …………… (a) X (b) Y (c) $X \times Y$ (d) $Y \times X$
2	The following figures shows four arrow diagrams one of them is not function it is ………. *(El-Dakahlia 2014)*  (a) X Y A B C 1 2 3 4 (b)  (c)  (d)
3	If f is a function from the set X to the set Y , then X is called ………. (a) the range of the function f (b) the domain of the function f (c) the codomain of the function f (d) the rule of the function f
4	If f is a function from the set X to the set Y , then Y is called ………. (a) the domain of the function. (b) the codomain of the function. (c) the range of the function. (d) the rule of the function.
5	If the relation $R = \{(4 , 3) , (1 , 3) , (2 , 5)\}$, then R represents a function where its range is ………. *(El-Kalyoubia 17)* (a) $\{1 , 2 , 4\}$ (b) $\{4 , 1 , 2 , 3 , 5\}$ (c) $\{3 , 5\}$ (d) $\mathbb{N}$
6	The opposite diagram represents a function on X , its range is ……….  *(Cairo 11)* (a) $\{a\}$ (b) $\{a , b , c\}$ (c) $\{a , b\}$ (d) $\{b , c\}$
7	The opposite figure represents a function on X , its range is ……….  (a) $\{1 , 0 , -1 , -2\}$ (b) $\{1 , 0 , -1\}$ (c) $\{0 , -1 , -2\}$ (d) $\{1 , -1 , -2\}$
8	If R is a function from X to Y where $X = \{2 , 4 , 5\}$, $Y = \{6 , 7\}$ and $R = \{(2 , 6) , (a , 6) , (5 , 6)\}$, then $a =$ ………. (a) 4 (b) 5 (c) 12 (d) 6

9	If : $(X - Y) \times Y = \{(1, 2), (1, 3)\}$ and $n(X \times Y) = 6$, then $X = \dots\dots$ (a) $\{1\}$ (b) $\{1, 2\}$ (c) $\{1, 3, 6\}$ (d) $\{1, 3, 2\}$
10	The point $(-3, 4)$ lies in $\dots\dots$ quadrant. (a) first (b) second (c) third (d) fourth
11	If the point $(3, b)$ lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 5X + 4$, then $b = \dots\dots$ (a) 5 (b) 4 (c) 0 (d) 19
12	The point A $(5, -3)$ lies on the $\dots\dots$ quadrant. (a) first (b) second (c) third (d) fourth
13	If the point $(5 - X, X - 4)$ lies in the fourth quadrant, then the value of $X = \dots\dots$ (a) 9 (b) 8 (c) 6 (d) 2
14	If the point $(X - 5, 3 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant, then X equals $\dots\dots$ (a) 2 (b) 3 (c) 4 (d) 5
15	If : $X = \{1, 3, 5\}$ and R is function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$, then the numerical value of the expression : $a + b = \dots\dots$ (a) 3 (b) 4 (c) 5 (d) 8
16	The opposite figure represents a function on X, its range = $\dots\dots$ (a) $\{a\}$ (b) $\{a, b, c\}$ (c) $\{a, b\}$ (d) $\{b, c\}$ 
17	If : $(a + 1, 5) = (-2, b - 1)$, then : $2a + b = \dots\dots$ *(El-Ismailia 2014)* (a) -12 (b) zero (c) 2 (d) 12

18 If : $n(X) = 3$, $Y = \{2 , 5\}$ then $n(X \times Y) =$ ....

(a) 2 (b) 3 (c) 5 (d) 6

19 If : $X = \{5 , 6 , 7\}$, then $n(X^2) =$ ....

(a) 3 (b) 6 (c) 9 (d) 12

20 If : $X \times Y = \{(6 , 3) , (6 , 4)\}$, then $n(X) =$ ....

(a) 3 (b) 1 (c) 4 (d) 2

21 The point $(-5 , 7)$ lies in .... quadrant.

(a) first (b) second (c) third (d) fourth

22 The point $(-2 , -5)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

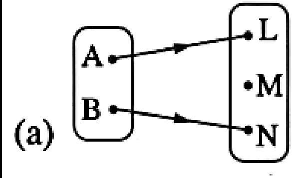
23 The point $(5 , -2)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

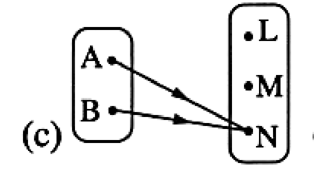
24 If the point $(x , 2)$ lies on y-axis, then $x =$ .... *(El-Fayoum 2011)*

(a) zero (b) 1 (c) 2 (d) 3

25 Which of the following relations does not represent a function from X to Y ? (Helwan 2011)

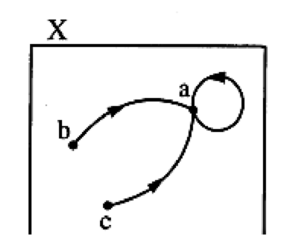
(a) 

(b) A L

(c) 

(d) A B L

26 The opposite diagram represents a function on X , its range is .... *(Cairo 2011)*



(a) $\{a\}$ (b) $\{a , b , c\}$

(c) $\{a , b\}$ (d) $\{b , c\}$

27 If : $X = \{3\}$, then : $X^2 =$ .... *(Cairo 2013)*

(a) 9 (b) $(3 , 3)$ (c) $\{9\}$ (d) $\{(3 , 3)\}$

EX. (2): Answer the following:

1	If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y, where "a R b" means "$a + b = 7$" for each of $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also by a Cartesian diagram. *(El-Menia 11 – Beni Suef 15 – Port Said 17)*
2	If $X = \{0, 1, 4, 7\}$, $Y = \{1, 3, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b < 8$" for each $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram and also by a Cartesian diagram. Is R a function ? And why ? *(El-Kalyoubia 11 – Alex. 18)*
3	If $X = \{2, 4, 5, 7\}$, $Y = \{4, 5, 6, 7, 9\}$ and R is a relation from X to Y where "a R b" means "$a \le b$" for each $a \in X$ and $b \in Y$. Write R and represent it by an arrow diagram and also by a Cartesian diagram.
4	If $X = \{-2, -1, 1, 2\}$, $Y = \{\frac{1}{8}, \frac{1}{3}, 1, 3, 8\}$ and R is a relation from X to Y, where "a R b" means "$a^3 = b$" for each $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and also Cartesian diagram.
5	If $X = \{2, 5, 8\}$ and $Y = \{10, 16, 24, 30\}$ and R is a relation from X to Y where "a R b" means "a is a factor of b" for each $a \in X$, $b \in Y$. Write R and represent it by an arrow diagram and by Cartesian diagram. Is R a function ? And why ?
6	If $X = \{2, 3, 4\}$, $Y = \{6, 8, 10, 11, 15\}$ and R is a relation from X to Y, where "a R b" means "a divides b" for each $a \in X$, $b \in Y$. Write the relation R
7	If $X = \{1, 2, 3, 6, 11\}$ and R is a relation on X where "a R b" means "$a + 2b$ = an odd number" for each $a \in X$, $b \in X$. Write R and represent it by an arrow diagram. Is R a function ? And why ?
8	If $X = \{1, 2, 4, 6, 10\}$ and R is a relation on X where "a R b" means "a is a multiple of b" for each $a \in X$, $b \in X$. Write R and represent it by an arrow diagram and also by a Cartesian diagram. Is R is a function ? And why ?

Explaining

Lesson (3)

The symbolic representation of the function - Polynomial functions

The symbolic representation of the function

- The function is usually denoted by one of the following letters. f or m or q or ...
  and the function f from the set X to the set Y is written mathematically as :

  $f : X \longrightarrow Y$ and is read as **f is a function from X to Y**

  or **m** : $X \longrightarrow Y$ and is read as **m is a function from X to Y** and so on ...

- If the ordered pair (x , y) belongs to the function , then the element y is called the image of the element x by the function f and we express it by one of the following two forms :

  $f : x \longmapsto y$ it is read as **f maps x to y**

  or $f : f(x) = y$ it is read as **f is a function where $f(x) = y$**

For example:

If $f : X \longrightarrow Y$ where $f : x \longmapsto x^2$, then $f : 3 \longmapsto 9$

, also can be written in the form : $f(x) = x^2$, hence $f(3) = 9$

Remark

The mathematical form $f(x) = x^2$ is called the rule of the function f, and it is used to find the image of any element of the domain by the function f

Remember that :

- **If f is a function from the set X to the set Y i.e. $f : X \longrightarrow Y$, then :**

1. X is called the **domain** of the function f
2. Y is called the **codomain** of the function f
3. The set of images of the elements of the set X by the function f is called the **range** of the function f which is a subset of the codomain Y

Polynomial functions

Definition

The function $f : R \longrightarrow R$, $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots\cdots + a_n x^n$
where a_0 , a_1 , a_2 , ... , $a_n \in \mathbb{R}$, $n \in \mathbb{N}$ is called a polynomial function.

i.e. The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (the index) of the variable x in any of its terms is a natural number.

For example: The following functions are all polynomial functions :

- $f : f(X) = 2X + 5$
- $g : g(X) = X^2 - 2X + 1$
- $k : k(X) = 8$
- $n : n(X) = 1 + \sqrt{2}X - 9X^3$

Remark

If the domain or the codomain of a function is not the set of real numbers , then that function is not a polynomial function.

For example:

- $f : f(X) = \sqrt{X}$ is not a polynomial function because $f(X)$ doesn't exist in $\mathbb{R}$ if X equals a negative number.

For example:

$f(-1) \notin \mathbb{R}$ because $\sqrt{-1} \notin \mathbb{R}$, so the domain of the function f is not the set of real numbers

- $h : h(X) = \frac{1}{X}$ is not a polynomial function

 because h (X) doesn't exist in $\mathbb{R}$ if X equals zero.

i.e. $h(0) \notin \mathbb{R}$

, so the domain of the function h is not the set of real numbers.

Remark

When we search if the function is a polynomial or not , we do not simplify its rule.

For example:

The function $f_1 : f_1(X) = X\left(X + \frac{1}{X}\right)$ doesn't represent a polynomial function

because $f_1(0) \notin \mathbb{R}$ while the function $f_2 : f_2(X) = X^2 + 1$ represents a polynomial function

And notice that: $X\left(X + \frac{1}{X}\right) = X^2 + 1$ for all real numbers except 0

The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function $f_1 : f_1(X) = 3X - \frac{1}{2}$ is of the first degree (a linear function)
- The function $f_2 : f_2(X) = \sqrt{5}X^2 - 3X + 4$ is of the second degree (quadratic function)
- The function $f_3 : f_3(X) = X^3 - 5X^2 + 4$ is of the third degree (cubic function)

Remark

The function $f : f(x) = a$ where $a \in \mathbb{R} - \{0\}$
is a polynomial function of zero degree. (a constant function) as $f(x) = 3$
In the case of $a = 0$ *i.e.* when $f(x) = 0$, then the function has no degree.

Example

If $f : \mathbb{R} \longrightarrow \mathbb{R}$, mention the degree of f in each of the following :

1. $f(x) = 5 - 3x$
2. $f(x) = 3x - x^2$
3. $f(x) = 5x - 3x^2 + x^3$
4. $f(x) = x^2(2 + x)^2$

Solution

1. f is of the first degree.
2. f is of the second degree.
3. f is of the third degree.
4. $\because f(x) = x^2(4 + 4x + x^2)$
   $= 4x^2 + 4x^3 + x^4$
   $\therefore f$ is a function of the fourth degree.

Notice that :

- When we want to determine the degree of the function we should simplify its rule to the simplest form before telling its degree.

Example

If $f : f(x) = x^2 - 2x + 5$

1. **Find :** $f(1)$, $f(0)$, $f(-2)$, $f\left(\frac{1}{2}\right)$ and $f\left(\sqrt{5}\right)$

Solution

1. $f(1) = (1)^2 - 2 \times (1) + 5 = 1 - 2 + 5 = 4$

   Similarly , $f(0) = 5$, $f(-2) = 13$
   , $f\left(\frac{1}{2}\right) = 4\frac{1}{4}$ and $f\left(\sqrt{5}\right) = 10 - 2\sqrt{5}$

Example

If $f(x) = 2x + b$ and $g(x) = x^2 + b$ and if $f(2) + g(-4) = 30$,
then find : $f(-2) - g(2)$

Solution

$\because f(2) = 2 \times 2 + b = 4 + b$, $g(-4) = (-4)^2 + b = 16 + b$

, $\because f(2) + g(-4) = 30$ $\therefore 4 + b + 16 + b = 30$

$\therefore 20 + 2b = 30$ $\therefore 2b = 30 - 20 = 10$

$\therefore b = \frac{10}{2} = 5$

$\therefore f(x) = 2x + 5$, $g(x) = x^2 + 5$

$\therefore f(-2) = 2 \times (-2) + 5 = 1$, $g(2) = 2^2 + 5 = 9$

$\therefore f(-2) - g(2) = 1 - 9 = -8$

Exercises

EX. (1): Choose the correct answer:

1	The function $f : f(x) = (x - 5)^3$ is a polynomial function of .... degree. (Qena 2011) (a) zero (b) second (c) third (d) fourth
2	The function : $f(x) = x(x^2 - 3)$ is of .... degree. (a) 1^{th} (b) 2^{th} (c) 3^{th} (d) 4^{th}
3	The function f where $f(x) = x^4 - 2x^3 + 5$ is a polynomial function of .... degree. (Cairo 2014) (a) first (b) second (c) third (d) fourth
4	The function f where $f(x) = 2x - 3x^4 + 1$ is a polynominal function of .... degree. (El-Sharkia 2011) (a) first (b) second (c) third (d) fourth
5	The function f where $f(x) = 6x^7 + 2x^5 - 4x + 1$ is a polynomial function of .... degree. (Kafr El-Sheikh 2011) (a) first (b) fifth (c) sixth (d) seventh
6	If : $f(x) = x + 3$, then $f(-2) =$ .... (a) 9 (b) -3 (c) 1 (d) 5

7 If : $f(X) = 5X - 7$, then $f(3) =$ ....

(a) 2 (b) 3 (c) 8 (d) 15

8 If : $f(X) = 5X - 3$, then $f(0) =$ ....

(a) 5 (b) 2 (c) 3 (d) -3

9 If : $f(X) = 7X - \frac{1}{2}$, then $f\left(\frac{1}{2}\right) =$ ....

(a) 7 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{7}{2}$

10 If : $f(X) = X^2 - \sqrt{2}\,X$, then : $f(\sqrt{2}) =$ .... (El-Dakahlia 2011)

(a) 4 (b) 2 (c) 6 (d) zero

11 If : $f(X) = X^2 - X + 3$, then : $f(3) =$ .... (Beni Suef 2011)

(a) 3 (b) 6 (c) 9 (d) 12

EX. (2): Answer the following:

1 If : $f(X) = 2X - 1$, prove that : $f(2) - 3f(1) =$ zero (El-Gharbia 2011)

2 If : $f(X) = 2X^2 - 5X + 2$, **then prove that :** $f(2) = f\left(\frac{1}{2}\right)$ *(Luxor 2014)*

3 If $f(X) = aX + b$, $f(a) = b$ Find the value of : $ab^2 + 5$ *(El-Sharkia 19)*

4 If $f(X) = X^2 - 2X - 5$, then prove that : $f\left(1 + \sqrt{6}\right) = f\left(1 - \sqrt{6}\right) = 0$

5 If $f(X) = X^2 - 3X$, $g(X) = X - 3$ *(El-Menia 17 – Alex.18 – Qena 19)*

1. Find : $f\left(\sqrt{2}\right) + 3g\left(\sqrt{2}\right)$
2. Prove that : $f(3) = g(3) = 0$

6 If $f : \mathbb{R} \longrightarrow \mathbb{R}$, mention the degree of f , then find $f(-2)$, $f(0)$, $f\left(\frac{1}{2}\right)$ when :

1. $f(X) = 3 - 2X$
2. $f(X) = X^2 - 4$

7 If the set of the function $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$ write :

1. The domain of the function f
2. The range of the function f
3. The rule of the function f

(Damietta 16 – North Sinai 17 – Luxor 19)

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8 **Which of the following functions represents a polynomial function :**

1. $f : f(x) = 2x - 5$
2. $f : f(x) = 3$
3. $f : f(x) = x + \frac{1}{x}$
4. 📖 $f : f(x) = x^3 + x^2 + 3$
5. 📖 $f : f(x) = x^2 + \sqrt{x} + 8$
6. 📖 $f : f(x) = x\left(x + \frac{1}{x} - 2\right)$
7. $f : f(x) = \sqrt[3]{x} + 8$
8. $f : f(x) = x(x^2 + x^{-2} - 4)$



Lesson (4)

The study of some polynomial functions

First The linear function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax + b$ where $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 2x + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 3x$

Notice that :

- In each of the shown functions , the index of x is 1 , therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is represented graphically by a straight line intersecting :
  - The y-axis at the point (0 , b)
  - The x-axis at the point $\left(\frac{-b}{a}, 0\right)$
- To represent a linear function , it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

Example

Graph each of the following linear functions :

1 $f : f(x) = 2x - 3$

2 $r : r(x) = -\frac{1}{2}x$

Solution

1 To graph this function :

- We determine 3 ordered pairs belonging to the function.

$\because f(x) = 2x - 3$

$\therefore f(-1) = 2(-1) - 3 = -5$ $\quad\therefore (-1 , -5) \in f$

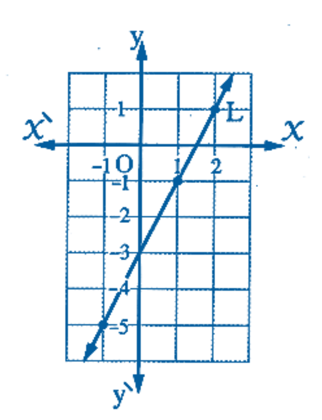
$, f(1) = 2 \times 1 - 3 = -1$ $\quad\therefore (1 , -1) \in f$

and $f(2) = 2 \times 2 - 3 = 1$ $\quad\therefore (2 , 1) \in f$

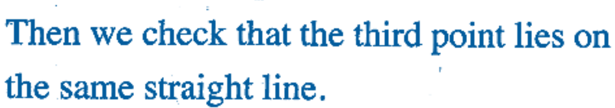
We can arrange these ordered pairs in the following table :

x	-1	1	2
$y = f(x)$	-5	-1	1

- We locate these three points which represents the three ordered pairs in the Cartesian plane and draw the straight line L which passes through any two points of them.



Then we check that the third point lies on the same straight line.



Then this straight line is the graphical representation of this function.

Notice that :

We can get the points of intersection of this straight line with the two axes and use them in representation.

- The point of intersection with y-axis = (0 , b) = (0 , – 3)
- The point of intersection with x-axis = $\left(-\frac{b}{a}, 0\right) = \left(\frac{3}{2}, 0\right)$

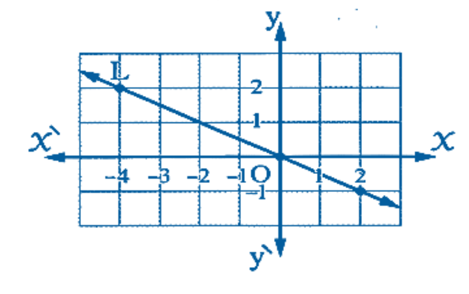
2 $\because r(x) = -\frac{1}{2}x \quad \therefore$

x	0	2	-4
$y = f(x)$	0	-1	2

Notice that :

If the coefficient of x is a fraction , it is better to choose numbers divisible by the denominator of this fraction to facilitate the representation.

From the opposite graph notice that , the straight line L passes through the origin point O (0 , 0)



Generally

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax$, $a \in \mathbb{R}^*$

is represented graphically by a straight line passing through the origin point (0 , 0)

Second The constant function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = b$, $b \in \mathbb{R}$ is called a constant function.

For example:

$f : f(x) = 5$ is a constant function where

$f(1) = 5$, $f(0) = 5$, $f(-2) = 5$, ... and so on.

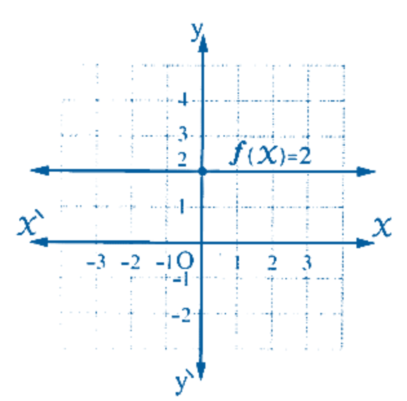
Graphical representation of the constant function

The constant function $f : f(x) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to x-axis and passes through the point $(0 , b)$ this line is :

- above x-axis if $b > 0$
- below x-axis if $b < 0$
- coincident with x-axis if $b = 0$

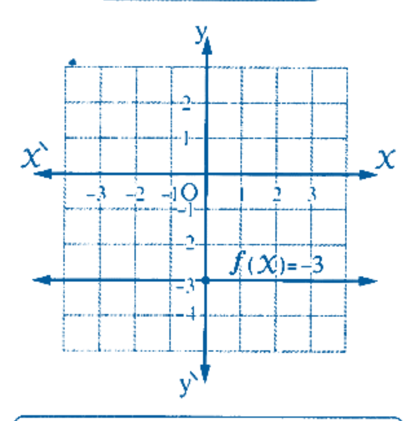
The following examples illustrate that :

$f : f(x) = 2$



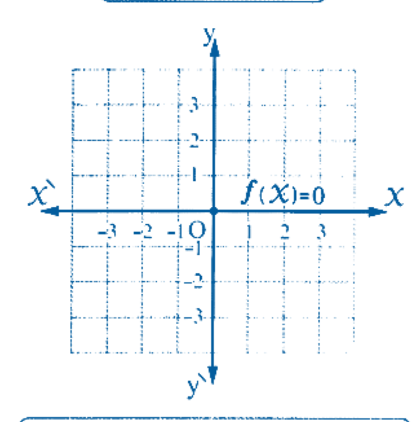
The straight line is above x-axis and passes through the point $(0 , 2)$

$f : f(x) = -3$



The straight line is below x-axis and passes through the point $(0 , -3)$

$f : f(x) = 0$



The straight line is coincident with x-axis and passes through the point $(0 , 0)$

Third The quadratic function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a\,x^2 + b\,x + c$
where a , b and c are real numbers , $a \neq 0$
is called a quadratic function (it is a polynomial function of the second degree).

Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 3\,x^2 - 7\,x + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 6 - x^2 + x$

Notice that :

In each of the shown functions , the highest index of x is 2 therefore each of them is a function of the 2^{nd} degree.

Example

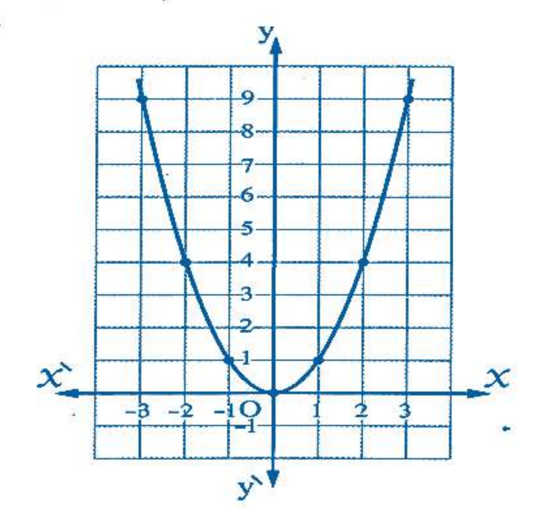
Graph each of the following quadratic functions :

1 $f : f(x) = x^2$ taking $x \in [-3 , 3]$

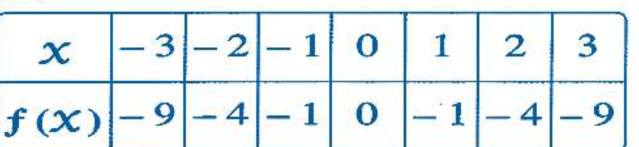
2 $f : f(x) = -x^2$ taking $x \in [-3 , 3]$

Solution

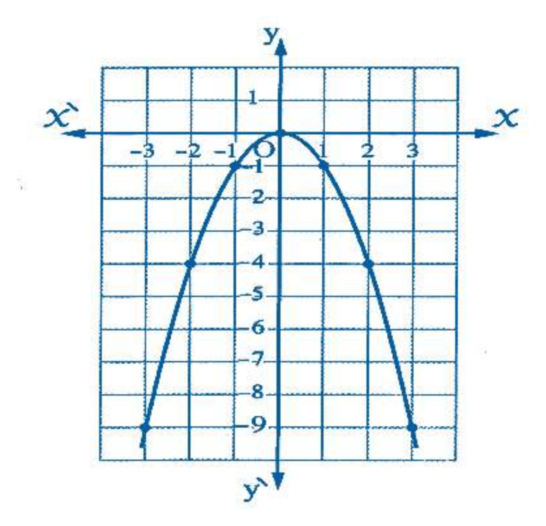
x	−3	−2	−1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9



Notice that : The coefficient of $x^2 > 0$



x	−3	−2	−1	0	1	2	3
$f(x)$	−9	−4	−1	0	−1	−4	−9



Notice that : The coefficient of $x^2 < 0$

1 The curve is symmetric with respect to y-axis *i.e.* the y-axis is the line of symmetry of the curve and its equation is $x = 0$

2 The point (0 , 0) is the point of the vertex of the curve , it is considered as a minimum value point of the curve because the whole curve lies up on it.

3 The minimum value of the function is zero it equals the y-coordinate of the vertex of the curve.

1 The curve is symmetric with respect to y-axis *i.e.* the y-axis is the line of symmetry of the curve and its equation is $x = 0$

2 The point (0 , 0) is the point of the vertex of the curve , it is considered as a maximum value point of the curve because the whole curve lies below it.

3 The maximum value of the function is zero it equals the y-coordinate of the vertex of the curve.

Generally , for any quadratic function

1 If the coefficient of x^2 is positive , then the curve is open upwards and the function has a minimum value point.

2 If the coefficient of x^2 is negative , then the curve is open downwards and the function has a maximum value point.

Example

Graph the function : $f : f(x) = x^2 - 2x - 3$ taking $x \in [-2 , 4]$

from the graph , find :

1 The point of the vertex of the curve.

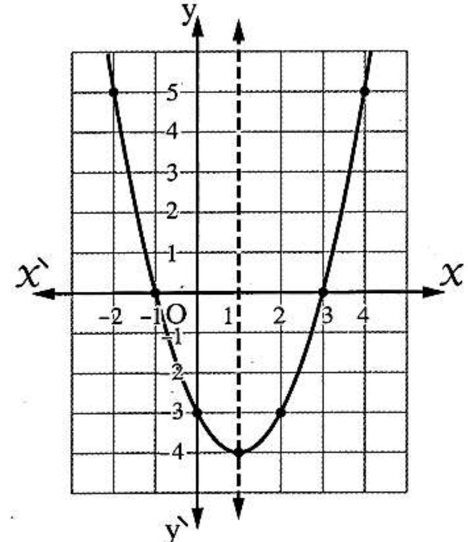
2 The equation of the line of symmetry.

3 The maximum or minimum value of the function.

Solution

$f(x) = x^2 - 2x - 3$

x	−2	−1	0	1	2	3	4
$f(x)$	5	0	−3	−4	−3	0	5



From the graph , we deduce that :

1 The vertex of the curve is $(1 , -4)$

2 The equation of the line of symmetry is $x = 1$, it is a straight line parallel to y-axis and passes through the vertex of the curve.

3 The minimum value of the function $= -4$

Example

Graph the function $f : f(x) = -x^2 + 3x + 2$ taking $x \in [-1 , 4]$ and from the graph , find :

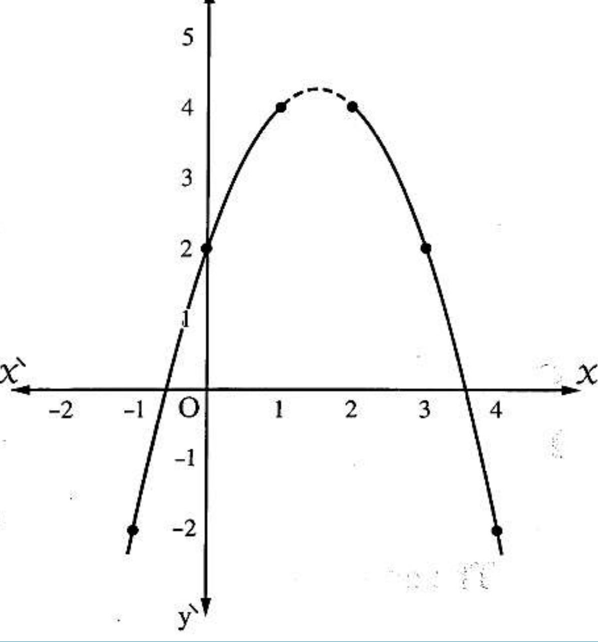
1 The maximum value or minimum value of the function.

2 The equation of the line of symmetry.

Solution

$f(x) = -x^2 + 3x + 2$

x	−1	0	1	2	3	4
$f(x)$	−2	2	4	4	2	−2



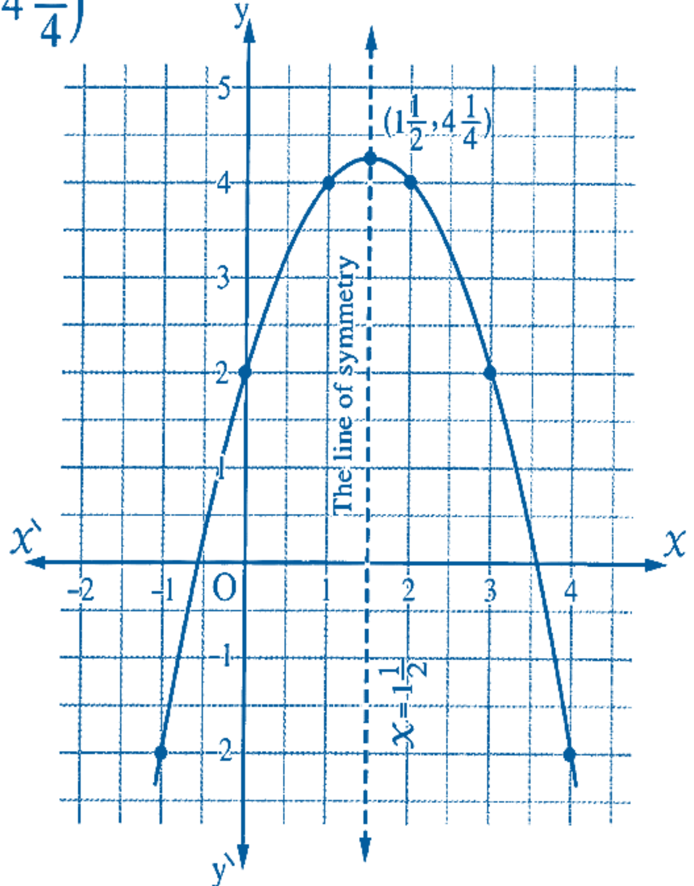
When we represent these ordered pairs , we notice that the point of the vertex of the curve is not among these points which makes the drawing.
of the dotted part in the opposite figure is inaccurate , so the studying of

the curve will be difficult , then we should find the vertex point of the curve algebraically as the following :

$\therefore$ The vertex of the curve is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



From the graph , we find that :



1 The maximum value of the function = $4\frac{1}{4}$

2 The equation of the line of symmetry is $X = 1\frac{1}{2}$

it is a straight line parallel to y-axis and passes through the vertex of the curve.

Finding the point of the vertex of the curve :

At the point of the vertex of the curve of the quadratic function , it will be :

- The X-coordinate $= \frac{-b}{2a}$
- The y-coordinate $= f\left(\frac{-b}{2a}\right)$

where b is the coefficient of X , a is the coefficient of X^2

$\therefore X$ at the vertex of the curve $= \frac{-3}{2 \times -1} = \frac{-3}{-2} = 1\frac{1}{2}$

$, \because f\left(1\frac{1}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 2 = 4\frac{1}{4}$

Exercises

EX. (1): Choose the correct answer:

1 If : $f(X) = 3$, then $f(2) =$ ……………
(a) 2 (b) 3 (c) 9 (d) 6

2 If : $f(X) = 5$, then $f(3) =$ ………. *(Beni Suef 2013)*
(a) 5 (b) 15 (c) 8 (d) $\frac{3}{5}$

3 If $f(X) = 7$, then $f(3) =$ ………. (Souhag 2011)
(a) 10 (b) 3 (c) 7 (d) 5

4 If $f(X) = 5$, then $f(3) - f(1) =$ ………. (Cairo 2006)
(a) $f(2)$ (b) 2 (c) zero (d) 10

5 If : $f(X) = 2$, then $f(3) - f(1) =$ ………. *(Dakahlia 2013)*
(a) $f(2)$ (b) 2 (c) zero (d) 10

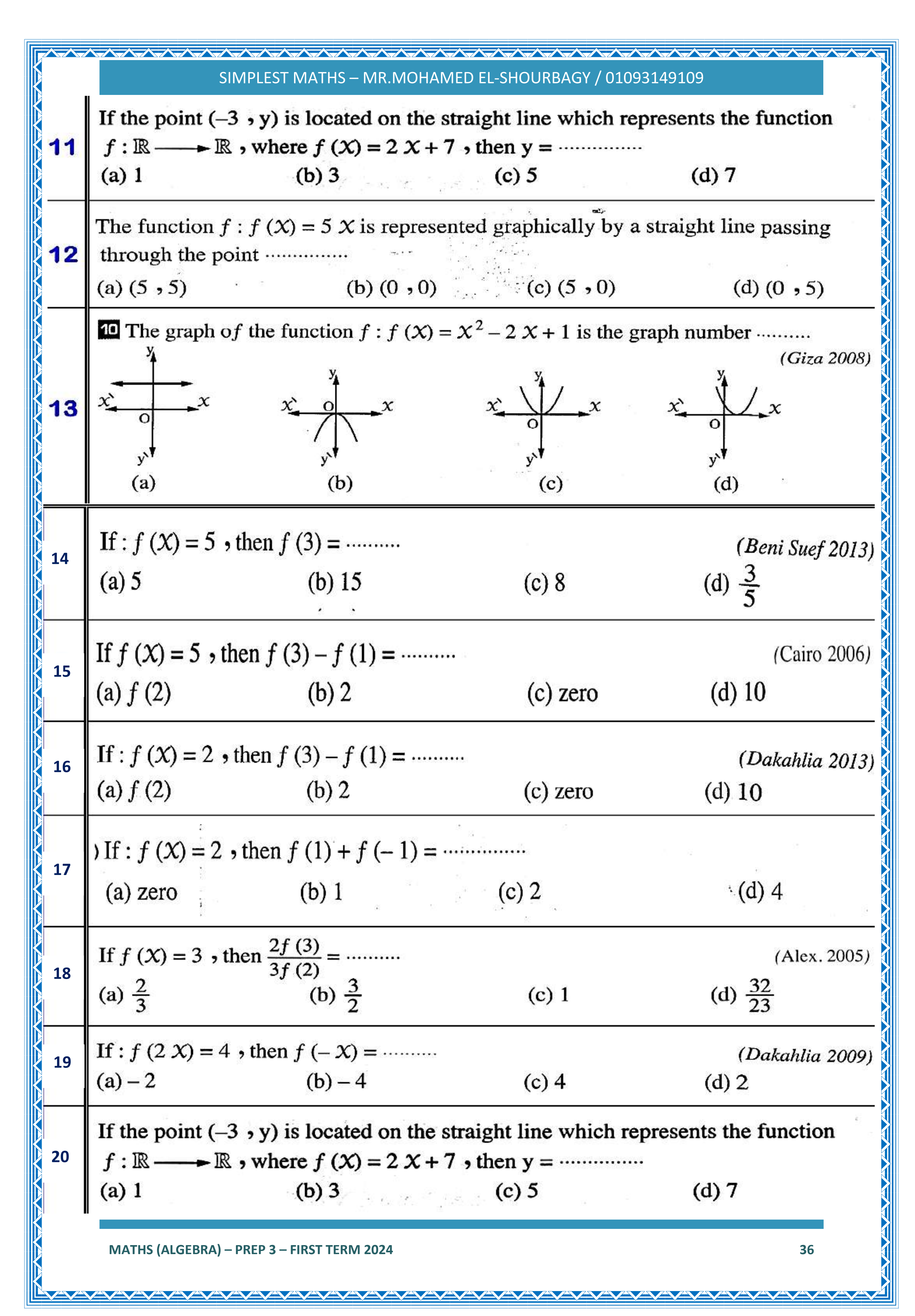
6 If : $f(X) = 2$, then $f(1) + f(-1) =$ ……………
(a) zero (b) 1 (c) 2 (d) 4

7 If $f(X) = 3$, then $\frac{2f(3)}{3f(2)} =$ ………. (Alex. 2005)
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

8 If : $f(2X) = 4$, then $f(-X) =$ ………. *(Dakahlia 2009)*
(a) -2 (b) -4 (c) 4 (d) 2

9 If the curve of the function f where $f(X) = X^2 - a$ passes through the point (1 , 0) , then a = ………. (Alex. 2011)
(a) ± 1 (b) -1 (c) 1 (d) zero

10 If the function f where $f(X) = 5X + 4$ is represented by a straight line passing through the point (3 , b), then b equals ……………
(a) 5 (b) 4 (c) 3 (d) 19



11 If the point $(-3 , y)$ is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 2X + 7$, then y = ...

(a) 1 (b) 3 (c) 5 (d) 7

12 The function $f : f(X) = 5X$ is represented graphically by a straight line passing through the point ....

(a) $(5 , 5)$ (b) $(0 , 0)$ (c) $(5 , 0)$ (d) $(0 , 5)$

13 The graph of the function $f : f(X) = X^2 - 2X + 1$ is the graph number .... *(Giza 2008)*

14 If : $f(X) = 5$, then $f(3) =$ .... *(Beni Suef 2013)*

(a) 5 (b) 15 (c) 8 (d) $\frac{3}{5}$

15 If $f(X) = 5$, then $f(3) - f(1) =$ .... (Cairo 2006)

(a) $f(2)$ (b) 2 (c) zero (d) 10

16 If : $f(X) = 2$, then $f(3) - f(1) =$ .... *(Dakahlia 2013)*

(a) $f(2)$ (b) 2 (c) zero (d) 10

17) If : $f(X) = 2$, then $f(1) + f(-1) =$ ....

(a) zero (b) 1 (c) 2 (d) 4

18 If $f(X) = 3$, then $\frac{2f(3)}{3f(2)} =$ .... (Alex. 2005)

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

19 If : $f(2X) = 4$, then $f(-X) =$ .... *(Dakahlia 2009)*

(a) -2 (b) -4 (c) 4 (d) 2

20 If the point $(-3 , y)$ is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, where $f(X) = 2X + 7$, then y = ...

(a) 1 (b) 3 (c) 5 (d) 7

EX. (2): Answer the following:

1 Represent the following functions graphically , where $x \in \mathbb{R}$:

[1] $f(x) = 5$ [2] $f(x) = -4$

[3] $f(x) = 0$ [4] $f(x) = 2\frac{1}{2}$

2 Represent each of the following linear functions graphically and find the points of intersection of the straight line which represents each of them with the coordinate axes , where $x \in \mathbb{R}$:

[1] $f : f(x) = x$ [2] $f : f(x) = -x$ [3] $f : f(x) = 3x$

3 If : $f(x) = 4x + b$, $f(3) = 15$, then find the value of : b

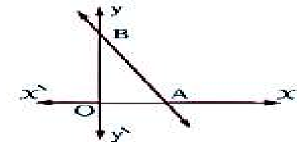
4 If : $f(x) = x^2 - x + 3$, find : $f(-2)$, $f(\text{zero})$, $f(\sqrt{3})$

5 The opposite figure represents the function f where $f(x) = 4 - 2x$

Find :

[1] The coordinates of A , B

[2] The area of $\triangle$ AOB



(Ismailia 16 – Luxor 19)

6 The opposite figure represents the curve of the function f where $f(x) = m - x^2$,

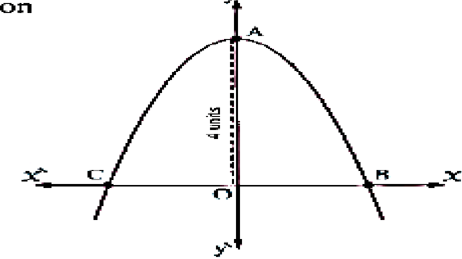
if OA = 4 units

Find : [1] The value of m

[2] The coordinates of B and C

[3] The area of the triangle with vertices


A , B and C



(North Sinai 16 – Luxor 18)

7 Represent each of the following functions graphically and from the graph , deduce the coordinates of the vertex of the curve and the equation of the line of symmetry and the maximum or minimum value of the function , where $x \in \mathbb{R}$:

[1] $f : f(x) = 2x^2$ taking $x \in [-2, 2]$

[2] $f : f(x) = x^2 + 1$ taking $x \in [-3, 3]$ (Beni Suef 14)

[3] $f : f(x) = x^2 - 2$ taking $x \in [-3, 3]$ (Cairo 15 – El-Beheira 17 – Port Said 18)

[4] $f : f(x) = 2 - x^2$ taking $x \in [-3, 3]$ (Giza 15 – Alex. 18 – El-Gharbia 19)

[5] $f : f(x) = x^2 - 2x$ taking $x \in [-2, 4]$ (Qena 11 – Cairo 18)

[6] $f : f(x) = x^2 + 2x + 1$ taking $x \in [-4, 2]$ (El-Sharkia 17 – El-Gharbia 18)

[7] $f : f(x) = (x - 2)^2$ taking $x \in [-1, 5]$ (Luxor 18 – Kafr El-Sheikh 19)



Unit 2

Lessons (1,2)
Ratio and Proportion

First : The Ratio : -

Generally

If a and b are two real numbers , then :
The ratio between a and b is written a : b or $\frac{a}{b}$ and is read a to b where :
a is called the antecedent of the ratio , b is called the consequent of the ratio, a and b are called together the two terms of the ratio.

Properties of the ratio

1. The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.
2. The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

First : The Proportion: -

Definition of proportion

It is the equality of two ratios or more.

i.e. If $\frac{a}{b} = \frac{c}{d}$, then the quantities a , b , c and d are proportional.
And vice versa : If a , b , c and d are proportional , then : $\frac{a}{b} = \frac{c}{d}$

- a is called the first proportional.
- b is called the second proportional.
- c is called the third proportional.
- d is called the fourth proportional.

a and d are called extremes and b and c are called means.

For example:

The numbers 1 , 4 , 7 and 28 are proportional numbers , because $\frac{1}{4} = \frac{7}{28}$
And : 1 is the first proportional, 4 is the second proportional , 7 is the third proportional , 28 is the fourth proportional, 1 and 28 are the extremes of this proportion and 4 and 7 are the means.

Properties of proportion

Property (1)

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (The product of the extremes = the product of the means)

Property (2)

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Also we can deduce that :

- If $a \times d = b \times c$, then $\frac{a}{c} = \frac{b}{d}$
- If $a \times d = b \times c$, then $\frac{b}{a} = \frac{d}{c}$
- If $a \times d = b \times c$, then $\frac{c}{a} = \frac{d}{b}$

Property (3)

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example:

If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Property (4)

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$ and $b = dm$ (where m is a constant $\neq 0$)

For example:

If $\frac{a}{b} = \frac{3}{4}$, then : $a = 3\,m$, $b = 4\,m$ (where m is a constant $\neq 0$)

Important remark

If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then

ⓐ $= bm$, ⓒ $= dm$

For example:

If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}\,b$, $c = \frac{3}{4}\,d$

Generally

If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = m$, then ⓐ = bm , ⓒ = dm , ⓔ = fm , ⋯

Property (5)

If we consider the proportion : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

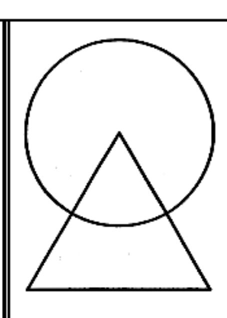
- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of given ratios.
- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15} = \frac{3}{5}$ = one of the given ratios.
- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

i.e. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m_3 , ⋯ are non-zero real numbers

, then $\frac{m_1 a + m_2 c + m_3 e + \cdots}{m_1 b + m_2 d + m_3 f + \cdots}$ = one of the given ratios

Examples:

1	If $\frac{x-2y}{x+3y} = \frac{1}{3}$, find : $\frac{y}{x}$
2	If $\frac{a}{b} = \frac{3}{4}$, then find the value of $\frac{4a+b}{2a-b}$
3	If $\frac{x}{y} = \frac{2}{3}$, then find the value of ratio : $\frac{3x+2y}{6y-x}$
4	If $\frac{a}{b} = \frac{3}{5}$, then find the value of 7 a + 9 b : 4 a + 2 b
5	If $\frac{21x+a}{7x+b} = \frac{a}{b}$, where $x \neq 0$ then find the value of : $\frac{a+2b}{2a}$
6	Prove that : a , b , c and d are proportional quantities if : $\frac{a+b}{b} = \frac{c+d}{d}$

7	Prove that : a , b , c and d are proportional quantities if : $\frac{a}{b-a} = \frac{c}{d-c}$
8	If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that : $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$
9	If : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, then prove that : $3X^2 + 3y^2 + z^2 = (2X + y)^2$
10	Find the number which if it is added to the two terms of the ratio 7:11 , it will be 2 : 3
11	Find the number that if we subtract thrice of it from each of the two terms of the Ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$
12	Find the number which if its square is added to each of the two terms of ratio 7 : 11 it becomes 4 : 5
13	Two integers , the ratio between them is 3 : 7 and if we subtracted 5 from each term , the ratio between each of them becomes 1 : 3, find the two numbers.
14	The ratio between two integers is $\frac{3}{4}$, if we add 4 to the small number and subtract 3 from the great number , the ratio will become 8 : 9 Find the two numbers.
15	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 Find the two numbers.
16	the opposite figure : Alaa shaded $\frac{5}{6}$ the area of the circle , $\frac{2}{3}$ the area of the triangle , find the ratio between the area of the circle and the area of the triangle. 

The solutions

Problem [1]

$\because 3x - 6y = x + 3y$ $\quad \therefore 2x = 9y$

$\therefore \frac{y}{x} = \frac{2}{9}$

Problem [2]

$\because \frac{a}{b} = \frac{3}{4}$ $\quad \therefore a = 3m, b = 4m$

(1) $\frac{4a+b}{2a-b} = \frac{12m+4m}{6m-4m} = \frac{16m}{2m} = 8$

Problem [3]

$\because \frac{x}{y} = \frac{2}{3}$ $\quad \therefore x = 2m, y = 3m$

$\therefore \frac{3x+2y}{6y-x} = \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}$

Problem [4]

$\because \frac{a}{b} = \frac{3}{5}$ $\quad \therefore a = 3m, b = 5m$

$\therefore \frac{7a+9b}{4a+2b} = \frac{21m+45m}{12m+10m} = \frac{66m}{22m} = 3$

Problem [5]

$\because \frac{21x+a}{7x+b} = \frac{a}{b}$ $\quad \therefore 21bx + ab = 7ax + ab$

$\therefore 21bx = 7ax$ $\quad \therefore 3b = a$

$\therefore \frac{a+2b}{2a} = \frac{3b+2b}{2 \times 3b} = \frac{5b}{6b} = \frac{5}{6}$

Problem [6]

$\because \frac{a+b}{b} = \frac{c+d}{d}$ $\quad \therefore d(a+b) = b(c+d)$

$\therefore ad + bd = bc + bd$ $\quad \therefore ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$

$\therefore$ a, b, c, d are proportional.

Problem [7]

$\because \frac{a}{b} = \frac{c}{d} = m$

$\therefore a = bm, c = dm$

$\therefore \frac{a}{b-a} = \frac{bm}{b-bm} = \frac{bm}{b(1-m)} = \frac{m}{1-m}$ (1)

, $\frac{c}{d-c} = \frac{dm}{d-dm} = \frac{dm}{d(1-m)} = \frac{m}{1-m}$ (2)

From (1) and (2) : $\therefore \frac{a}{b-a} = \frac{c}{d-c}$

Problem [8]

$\because \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore x = 3m, y = 4m, z = 5m$

$\therefore$ L.H.S. $= \frac{2y-z}{3x-2y+z} = \frac{8m-5m}{9m-8m+5m}$

$= \frac{3m}{6m} = \frac{1}{2} =$ R.H.S.

Problem [9]

$\because \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore x = 3m, y = 4m, z = 5m$

$\therefore 3x^2 + 3y^2 + z^2$

$= 3 \times 9m^2 + 3 \times 16m^2 + 25m^2$

$= 27m^2 + 48m^2 + 25m^2 = 100m^2$ (1)

, $(2x+y)^2 = (6m+4m)^2 = (10m)^2 = 100m^2$ (2)

From (1) and (2) : $\therefore 3x^2 + 3y^2 + z^2 = (2x+y)^2$

Problem [10]

Let the number be x $\quad \therefore \frac{7+x}{11+x} = \frac{2}{3}$

$\therefore 21 + 3x = 22 + 2x$ $\quad \therefore x = 1$

$\therefore$ The required number is 1

Problem [11]

Let the number be x

$\therefore \frac{49-3x}{69-3x} = \frac{2}{3}$

$\therefore 147 - 9x = 138 - 6x$

$\therefore 3x = 9$

$\therefore x = 3$

$\therefore$ The required number $= 3$

Problem [12]

Let the number be x

$\therefore \frac{7+x^2}{11+x^2} = \frac{4}{5}$

$\therefore 35 + 5x^2 = 44 + 4x^2$

$\therefore x^2 = 9$

$\therefore x = \pm 3$

$\therefore$ The required number is 3 or -3

Problem [13]

Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{3}{7}$

$\therefore a = 3m$, $b = 7m$

$\therefore \frac{3m-5}{7m-5} = \frac{1}{3}$

$\therefore 9m - 15 = 7m - 5$

$\therefore 2m = 10$

$\therefore m = 5$

$\therefore$ The two numbers are 15 and 35

Problem [14]

Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{3}{4}$

$\therefore a = 3m$, $b = 4m$

$\therefore \frac{3m+4}{4m-3} = \frac{8}{9}$

$\therefore 27m + 36 = 32m - 24$

$\therefore 5m = 60$

$\therefore m = 12$

$\therefore$ The two numbers are 36 and 48

Problem [15]

Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{2}{3}$

$\therefore a = 2m$, $b = 3m$

$\therefore \frac{2m+7}{3m-12} = \frac{5}{3}$

$\therefore 6m + 21 = 15m - 60$

$\therefore 81 = 9m$

$\therefore m = 9$

$\therefore$ The two numbers are 18 and 27

Problem [16]

$\because$ The area of the unshaded part from the circle

$= 1 - \frac{5}{6} = \frac{1}{6}$ of the area of the circle

, the area of the unshaded part from the triangle

$= 1 - \frac{2}{3} = \frac{1}{3}$ of the area of the triangle.

$\therefore \frac{1}{6}$ of the area of the circle

$= \frac{1}{3}$ of the area of the triangle

$\therefore$ The area of the circle : the area of the triangle

$= \frac{1}{3} : \frac{1}{6}$ (multiply by 6) $= 2 : 1$

EX. (1): Choose the correct answer:

Exercises

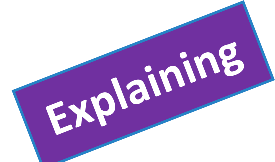
1	If : 24 , X , 6 and 3 are proportional quantities , then X = ... A) 9 B) 12 C) 18 D) 48
2	The fourth proportional for the 2 , 6 , 9 is .... A) 12 B) 18 C) 27 D) 54
3	The fourth proportional for the 3 , 6 , 6 is........................ A) -12 B) 6 C) 9 D) 12
4	If $\frac{a}{b} = \frac{3}{2}$, then $\frac{a+b}{a-b}$ = ... A) $\frac{3}{2}$ B) 5 C) $\frac{4}{5}$ D) 2
5	If : $\frac{a}{b} = \frac{3}{4}$, then 4 a – 3 b + 5 =........................ A) 0 B) 1 C) 3 D) 5
6	If : $\frac{a}{b} = \frac{5}{3}$, then $\frac{3a}{5b}$ = ... A) 1 B) $\frac{5}{3}$ C) 3 D) 5
7	If : $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $\frac{a+c}{b+d}$ = ... A) $\frac{3}{4}$ B) $\frac{7}{4}$ C) $\frac{3}{7}$ D) $\frac{9}{16}$
8	If : $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a}$ = ... A) $\frac{1}{5}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{3}{5}$
9	If : $\frac{a}{12} = \frac{b}{5} = \frac{a-2b}{k}$, then k =........................ A) 1 B) 2 C) 3 D) 4
10	If : $\frac{a}{5} = \frac{b}{4} = \frac{a+b}{k}$, then k = ... A) 5 B) 4 C) 9 D) 1
11	If : $\frac{X}{Y} = \frac{Z}{1}$ which of the following is right .... A) $\frac{X}{1} = \frac{Y}{Z}$ B) $\frac{X}{Z} = \frac{1}{Y}$ C) $\frac{X}{Y} = \frac{1}{Z}$ D) $\frac{X}{Z} = \frac{Y}{1}$
12	If : $\frac{X}{2} = \frac{Y}{7} = \frac{X+Y}{K}$, then K =........................ A) 4 B) 10 C) 6 D) 9
13	4 X = 25 Y , then $\frac{X}{Y}$ = ... A) $\frac{4}{25}$ B) $\frac{2}{5}$ C) $\frac{5}{2}$ D) $\frac{25}{4}$
14	If : A , Y , B and 2 X are proportional, then : $\frac{A}{B}$ = ... A) 2 : 1 B) 1 : 2 C) 1 : 3 D) 1 : 4

15	If : $\frac{3a}{5b} = \frac{1}{2}$, then : $\frac{a}{b}$ = ... A) $\frac{6}{5}$ B) $\frac{5}{6}$ C) $\frac{2}{3}$ D) $\frac{3}{2}$
16	If : $\frac{a+b}{5} = \frac{a-b}{3}$, then : $\frac{a}{b}$ = ... A) $\frac{1}{2}$ B) 2 C) 4 D) $\frac{1}{4}$
17	If : 2 a = 3 b , then $\frac{5b}{a}$ = ... A) $\frac{5}{3}$ B) $\frac{5}{2}$ C) $\frac{15}{2}$ D) $\frac{10}{3}$
18	If 3 x = 5 y , then $\frac{5y}{3x}$ = ... A) 1 B) 2 C) $\frac{3}{5}$ D) $\frac{5}{3}$
19	If : 4 x = 5 y, then : $\frac{5y}{4x}$ =.......................... A) 1 B) 2 C) 3 D) 4

EX. (2): Answer the following:

1	If $\frac{x}{y} = \frac{2}{3}$, find the value of the ratio : $\frac{3x+2y}{6y-x}$ *(Souhag 19)* « $\frac{3}{4}$ »
2	**Prove that : a , b , c and d are proportional quantities if :** 1 $\frac{a+b}{b} = \frac{c+d}{d}$
3	If $\frac{a}{b} = \frac{3}{5}$, then find the value of 7 a + 9 b : 4 a + 2 b
4	Find the number which if it is added to the two terms of the ratio 7 : 11 it will be 2 : 3 *(Alex. 14 – Cairo 17 – El-Fayoum 18 – Giza 19)* « 1 »
5	Find the number that if we subtract thrice of it from each of the two terms of the ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$ *(Giza 12)* « 3 »
6	Find the number which if its square is added to each of the two terms of ratio 7 : 11 it becomes 4 : 5 *(Cairo 11 – Suez 17)* « 3 or – 3 »

7	Find the positive number which if we add its square to each of the two terms of ratio 5 : 11 it becomes 3 : 5 *(Kafr El-Sheikh 17 – Matrouh 17 – Giza 19)* « 2 »
8	Two integers , the ratio between them is 3 : 7 and if we subtracted 5 from each term , the ratio between each of them becomes 1 : 3 , find the two numbers. *(New Valley 13 – Ismailia 17 – Alex. 18)* « 15 , 35 »
9	Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3 Find the two numbers. *(El-Beheira 15 – Beni Suef 17 – Matrouh 18)* « 18 , 27 »
10	If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$, prove that : $2a - 5b + 3c$ = one of the given ratios.
11	**If a , b , c and d are proportional quantities , prove that :** 1 $\frac{3a + c}{5a - 2c} = \frac{3b + d}{5b - 2d}$ *(Assiut 17)* 2 $\frac{3a - 2c}{5a + 3c} = \frac{3b - 2d}{5b + 3d}$ *(Suez 16 – Kafr El-Sheikh 18)*
12	If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$, then find the value of : x *(El-Gharbia 16 – Qena 17 – Luxor 18 – Aswan 19)* « 7 »
13	If $\frac{y}{x - z} = \frac{x}{y} = \frac{x + y}{z}$, prove that each ratio is equal to 2 (unless $x + y = 0$) , then find $x : y : z$ *(El-Beheira 18)* « 4 : 2 : 3 »
14	**If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, prove that :** 1 $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$ *(Suez 14 – Giza 15 – North Sinai 18 – Port Said 19)* 2 $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$ *(Souhag 16 – El-Menia 12 – Damietta 19)*
15	If $\frac{x}{2a + b} = \frac{y}{2b - c} = \frac{z}{2c - a}$, then prove that : $\frac{2x + y}{4a + 4b - c} = \frac{2x + 2y + z}{3a + 6b}$ *(El-Beheira 17 – El-Kalyoubia 18 – Matrouh 19)*
16	**If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that :** 1 $\frac{a + 5c}{b + 5d} = \frac{c - 3e}{d - 3f}$ 2 $\frac{2a + 7c - 4e}{2b + 7d - 4f} = \frac{a - 8e}{b - 8f}$



Lessons (3)
Continued Proportion

Definition :

The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

In this proportion , a is called the **first proportion** , c is called the **third proportion** and b is called the **middle proportion (proportional mean)**.

For Example : -

The numbers 4 , 6 and 9 form a continued proportion because : $\frac{4}{6} = \frac{6}{9}$ or because : $(6)^2 = 4 \times 9$ where 6 is the middle proportion , 4 is the first proportion and 9 is the third proportion.

Notice That : -

1. If a , b and c are in continued proportion , then : $b^2 = a\,c$ *i.e.* $b = \pm\sqrt{ac}$ and the two quantities a and c should be either both positive or both negative.
2. For any two positive numbers or any two negative numbers x and y , there are two middle proportions ($\sqrt{x\,y}$ and $-\sqrt{x\,y}$)

Remark : -

If a , b and c are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = m$

, then $\frac{b}{c} = m$ $\therefore$ (b) $= cm$ (1)

, $\because \frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (1) : $\therefore a = (cm)\,m$ $\therefore$ (a) $= cm^2$

i.e. If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

General Definition : -

The quantities a , b , c , d , ... are in continued proportion if : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \cdots$

For Example : -

The numbers 16 , 24 , 36 and 54 are in continued proportion

because : $\frac{16}{24} = \frac{24}{36} = \frac{36}{54}$, each ratio $= \frac{2}{3}$

Remark : -

If a , b , c and d are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then :

$\frac{c}{d} = m$ $\qquad \therefore \boxed{c} = dm$ $\qquad (1)$

, $\frac{b}{c} = m$ $\qquad \therefore b = cm$

Substituting for c from (1) : $\therefore b = (dm)\, m$ $\qquad \therefore \boxed{b} = dm^2$ $\qquad (2)$

, $\frac{a}{b} = m$ $\qquad \therefore a = bm$

Substituting for b from (2) : $\therefore a = (dm^2)\, m$ $\qquad \therefore \boxed{a} = dm^3$

i.e. If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\boxed{c = dm}$, $\boxed{b = dm^2}$ and $\boxed{a = dm^3}$

Examples:

1	**Find the middle proportion between -2 and -8**
2	**Find the third proportion for 3 and 6**
3	**If b is the middle proportion between a and c , prove that : $\frac{a}{c} = \frac{b^2}{c^2}$**
4	**If b is the middle proportion between a and c , prove that : $\frac{a+b}{b+c} = \frac{a}{b}$**
5	**If b is the middle proportion between a and c , prove that : $\frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{c\,b}$**
6	**If b is the middle proportion between a and c , prove that : $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2\,a}{c}$**
7	**If a , b , c and d are in continued proportion prove that : $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$**
8	**If a , b , c and d are in continued proportion prove that : $\frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$**
9	**If a , b , c and d are in continued proportion prove that : $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$**
10	**If a , b , c and d are in continued proportion prove that : $\frac{a^2+d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$**
11	**If $\frac{a^2+d^2}{c(a+c)} = \frac{a^2+d^2}{c(a+c)}$, prove that b is the middle proportion between a and c where ac is a positive quantity.**

The solutions

Problem [1]

The middle proportion $= \pm\sqrt{-2 \times -8}$

$= \pm\sqrt{16} = \pm 4$

Problem [2]

third $6^2 \div 3 = 12$

Problem [3]

Let $\frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore b = cm \quad , a = cm^2$

$\therefore \frac{a-b}{b-c} = \frac{cm^2 - cm}{cm - c} = \frac{cm(m-1)}{c(m-1)} = m \quad (1)$

$, \frac{a+3b}{3c+b} = \frac{cm^2 + 3cm}{3c + cm} = \frac{cm(m+3)}{c(3+m)} = m \quad (2)$

From (1) and (2) : $\quad \therefore \frac{a-b}{b-c} = \frac{a+3b}{3c+b}$

Problem [4]

Let $\frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore b = cm \quad , a = cm^2$

$\therefore \frac{a+b}{b+c} = \frac{cm^2 + cm}{cm + c} = \frac{cm(m+1)}{c(m+1)} = m \quad (1)$

$, \frac{a}{b} = \frac{cm^2}{cm} = m \quad (2)$

From (1) and (2) : $\quad \therefore \frac{a+b}{b+c} = \frac{a}{b}$

Problem [5]

Let $\frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore b = cm \quad , a = cm^2$

$\therefore \frac{a^3+b^3}{b^3+c^3} = \frac{c^3m^6 + c^3m^3}{c^3m^3 + c^3} = \frac{c^3m^3(m^3+1)}{c^3(m^3+1)} = m^3 \quad (1)$

$, \frac{a^2}{cb} = \frac{c^2m^4}{c \times cm} = m^3 \quad (2)$

From (1) and (2) : $\quad \therefore \frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{cb}$

Problem [6]

Let $\frac{a}{b} = \frac{b}{c} = m$ $\quad \therefore b = cm \quad , a = cm^2$

$\therefore \frac{2a}{c} = \frac{2cm^2}{c} = 2m^2 \quad (1)$

$, \frac{a^2}{b^2} + \frac{b^2}{c^2} = m^2 + m^2 = 2m^2 \quad (2)$

From (1) and (2) : $\quad \therefore \frac{2a}{c} = \frac{a^2}{b^2} + \frac{b^2}{c^2}$

Another solution : $\because b^2 = ac$

$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2a}{c} =$ R.H.S.

Problem [7]

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \quad , b = dm^2 \quad , a = dm^3$

$\therefore \frac{ab - cd}{b^2 - c^2} = \frac{dm^3 \times dm^2 - dm \times d}{d^2m^4 - d^2m^2} = \frac{d^2m^5 - d^2m}{d^2m^2(m^2-1)}$

$= \frac{d^2m(m^4-1)}{d^2m^2(m^2-1)} = \frac{(m^2-1)(m^2+1)}{m(m^2-1)} = \frac{m^2+1}{m} \quad (1)$

$, \frac{a+c}{b} = \frac{dm^3 + dm}{dm^2} = \frac{dm(m^2+1)}{dm^2} = \frac{m^2+1}{m} \quad (2)$

From (1) and (2) : $\quad \therefore \frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

Problem [8]

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \quad , b = dm^2 \quad , a = dm^3$

$\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{d^2m^6 + d^2m^4 + d^2m^2}{d^2m^4 + d^2m^2 + d^2}$

$= \frac{d^2m^2(m^4+m^2+1)}{d^2(m^4+m^2+1)} = m^2 \quad (1)$

$, \frac{ac}{bd} = \frac{dm^3 \times dm}{dm^2 \times d} = m^2 \quad (2)$

From (1) and (2) : $\quad \therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$

Problem [9]

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \quad , b = d\,m^2 \quad , a = d\,m^3$

$\therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5d^3m^9 - 3d^3m^3}{5d^3m^6 - 3d^3}}$

$= \sqrt[3]{\frac{d^3m^3(5m^6 - 3)}{d^3(5m^6 - 3)}} = \sqrt[3]{m^3} = m \qquad (1)$

$, \frac{a+c}{b+d} = \frac{dm^3 + dm}{dm^2 + d} = \frac{dm(m^2+1)}{d(m^2+1)} = m \qquad (2)$

From (1) and (2) : $\quad \therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$

Problem [10]

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \quad , b = d\,m^2 \quad , a = d\,m^3$

$\therefore \frac{a^2 + d^2}{c(a+c)} = \frac{d^2m^6 + d^2}{dm(dm^3 + dm)} = \frac{d^2(m^6+1)}{d^2m^2(m^2+1)}$

$= \frac{(m^2+1)(m^4 - m^2 + 1)}{m^2(m^2+1)} = \frac{m^4 - m^2 + 1}{m^2} \qquad (1)$

$, \frac{b}{d} + \frac{d}{b} - 1 = \frac{dm^2}{d} + \frac{d}{dm^2} - 1 = m^2 + \frac{1}{m^2} - 1$

$= \frac{m^4 + 1 - m^2}{m^2} = \frac{m^4 - m^2 + 1}{m^2} \qquad (2)$

From (1) and (2) : $\quad \therefore \frac{a^2 + d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$

Problem [11]

If $\frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$, prove that b is the middle proportion between a and c where ac is a positive quantity.

$\because \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2} \qquad \therefore a^2c^2 + b^2c^2 = b^4 + b^2c^2$

$\therefore a^2c^2 = b^4 \qquad \therefore ac = b^2$

EX. (1): Choose the correct answer:

Exercises

1 The third proportion of the two numbers 9 and – 12 is .... *(El-Beheira 11)*

(a) – 16 (b) 8 (c) 16 (d) 108

2 The proportional mean between x and y is .... *(South Sinai 17)*

(a) $\sqrt{xy}$ (b) $-\sqrt{xy}$ (c) $\pm\sqrt{xy}$ (d) xy

3 If ℓ , m , z are in continued proportion , then ℓ = ...

(a) $\pm\sqrt{mz}$ (b) mz (c) $\frac{m}{z}$ (d) $\frac{m^2}{z}$

4 If the number 6 is the positive proportional mean of the two numbers 2 and m ,

then m = ... *(Aswan 13)*

(a) 8 (b) 12 (c) 18 (d) 36

5 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$, then a = ... *(El-Monofia 12)*

(a) 5×2^2 (b) 40 (c) 10 (d) 2×5^3

6 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$, then $\frac{a}{d}$ = ... *(El-Sharkia 13)*

(a) 2 (b) 4 (c) 8 (d) 16

7 If $6a^2b^2$, 3 a b and c are proportional quantities , then c = ...

(a) – 3 (b) 3 a b (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

8 If a , 2 , 4 , b are in continued proportion , then a + b = ... *(Beni Suef 08)*

(a) 8 (b) 1 (c) 9 (d) 7

9 The proportional mean between $(x-2)$ and $(x+2)$ is .... *(Cairo 09)*

(a) $\sqrt{x+2}$ (b) x^2-4 (c) $\pm\sqrt{x^2-4}$ (d) $\sqrt{x^2-4}$

10 The number which is added to each of the numbers 1 , 3 and 6 to become in continued proportion is .... *(Damietta 13)*

(a) 1 (b) 2 (c) 3 (d) 6

EX. (2): Answer the following:



1 **If b is the middle proportion between a and c , prove that :**

1. $\frac{a}{c} = \frac{b^2}{c^2}$ *(Giza 14)*
2. $\frac{2a + 3b}{2b + 3c} = \frac{a}{b}$ *(Ismailia 17)*
3. $\frac{a - b}{b - c} = \frac{a + 3b}{3c + b}$
4. $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$ *(Cairo 17 – El-Kalyoubia 18)*
5. $\left(\frac{b - c}{a - b}\right)^2 = \frac{c}{a}$
6. $\frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{cb}$ *(El-Monofia 11)*
7. $\frac{a^3 - 4b^3}{b^3 - 4c^3} = \frac{b^3}{c^3}$
8. $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$
9. $\frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{a^2 - b^2}{b^2 - c^2}$
10. $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$ *(Aswan 16 – Port Said 17 – El-Dakahlia 19)*
11. $\frac{a + b + c}{a^{-1} + b^{-1} + c^{-1}} = b^2$
12. $\frac{ac}{b(b + c)} = \frac{a}{a + b}$ *(El-Gharbia 17)*

2 **If a , b , c and d are in continued proportion , prove that :**

1. $\frac{a - 2b}{b - 2c} = \frac{3b + 4c}{3c + 4d}$
2. $\frac{3a + 5c}{3b + 5d} = \frac{a - 4c}{b - 4d}$
3. $\frac{3a - 5c}{a - b + c} = \frac{3b - 5d}{b - c + d}$
4. $\frac{a - d}{a + b + c} = \frac{a - 2b + c}{a - b}$
5. $\frac{c^2 - d^2}{a - c} = \frac{bd}{a}$ *(Qena 15 – Matrouh 17 – El-Beheira 18)*
6. $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$ *(El-Beheira 15 – Alex. 17 – Beni Suef 18)*
7. $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$ *(Qena 16 – El-Monofia 17)*
8. $\frac{a}{b + d} = \frac{c^3}{c^2 d + d^3}$ *(Alex. 19)*
9. $\frac{a^2 + b^2 + c^2}{b^2 + c^2 + d^2} = \frac{ac}{bd}$ *(El-Dakahlia 11)*
10. $\frac{2a + 3d}{3a - 4d} = \frac{2a^3 + 3b^3}{3a^3 - 4b^3}$
11. $\frac{a + 5b}{b + 5c} = \sqrt{\frac{b}{d}}$

3 If 5 a , 6 b , 7 c and 8 d are positive quantities in continued proportion ,

prove that : $\sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a + 6b}{7c + 8d}}$



Lessons (4)

Direct variation & Inverse variation

First The direct variation

Definition

It is said that y varies directly as x and it is written $y \propto x$ if $y = m\,x$

i.e. $\frac{y}{x} = m$ (where m is a constant $\neq 0$)

If the variable x took the two values x_1 and x_2 and y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

Second The inverse variation

Definition

It is said that y varies inversely as x and it is written $y \propto \frac{1}{x}$ if $y = \frac{m}{x}$

i.e. $x\,y = m$, where (m is a constant $\neq 0$)

If the variable x took the two values x_1 , x_2 and as a result for that y took the two values y_1 and y_2 respectively , then : $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

Examples:

1	If : $y \propto \frac{1}{X}$ and y = 3 when X = 2 , Find : 1) the relation between X and y 2) the value of y when X = 1.5
2	If : $y^2 \propto X^3$, Find the relation between X and y where y = 3 as X = 2
3	If : $\frac{a+b}{3} = \frac{2b+c}{6}$, then prove that : $c \propto a$
4	If : $X^2 y^2 - 6Xy + 9 = 0$, then prove that : y varies inversely as X
5	If : X = z + 8 and z varies inversely as y and z = 2 as y = 3 , Find y as X = 3
6	If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and h = 27 cm. when r = 10.5 , Find h when r = 15.75 cm.
7	If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) If the body weighs 84 kg. , on the ground and its weight on the moon is 14 kg. What will its weight be on the moon if its weight on the ground is 144 kg?
8	If the value of speed v that water passes through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and v = 5 cm./s. when r = 3 cm., find v when r = 2.5 cm.

The solutions

Problem [1]

(1) $\because y \propto x$ $\quad \therefore y = m x$

$\therefore 14 = 42 m$ $\quad \therefore m = \frac{1}{3}$

$\therefore y = \frac{1}{3} x$ (The relation between x , y)

(2) As $x = 60$ $\quad \therefore y = \frac{1}{3} \times 60 = 20$

Problem [2]

$\because y \propto x^2$ $\quad \therefore y = m x^2$

$\therefore 4 = m (3)^2$ $\quad \therefore m = \frac{4}{9}$

$\therefore y = \frac{4}{9} x^2$ (The relation between x and y)

As $x = 9$ $\quad \therefore y = \frac{4}{9} \times 9^2 = 36$

Problem [3]

$\because y \propto (x + 1)$ $\quad \therefore y = m (x + 1)$

$\because y = 2 , x = 3$ $\quad \therefore 2 = m (3 + 1)$

$\therefore m = \frac{1}{2}$ $\quad \therefore y = \frac{1}{2} (x + 1)$

Problem [4]

$\because \frac{21 x - y}{7 x - z} = \frac{y}{z}$ $\quad \therefore 21 x z - z y = 7 x y - z y$

$\therefore 21 x z = 7 x y$ $\quad \therefore 3 z = y$ $\quad \therefore y \propto z$

Problem [5]

$\because x^4 y^2 - 14 x^2 y + 49 = 0$

$\therefore (x^2 y - 7)^2 = 0$ $\quad \therefore x^2 y - 7 = 0$

$\therefore x^2 y = 7$ $\quad \therefore y \propto \frac{1}{x^2}$

Problem [6]

$\because y = a - 9$ $\quad , y \propto \frac{1}{x^2}$ $\quad \therefore y = \frac{m}{x^2}$

$\therefore \frac{m}{x^2} = a - 9$ $\quad \therefore m = x^2 (a - 9)$

$\therefore a = 18$ as $x = \frac{2}{3}$ $\quad \therefore m = \frac{4}{9}(18 - 9)$

$\therefore m = \frac{4}{9} \times 9 = 4$ $\quad \therefore y = \frac{4}{x^2}$

As $x = 1$ $\quad \therefore y = 4$

Problem [7]

$\because d \propto t$ $\quad \therefore \frac{d_1}{d_2} = \frac{t_1}{t_2}$

$\therefore \frac{150}{d_2} = \frac{6}{10}$ $\quad \therefore d_2 = \frac{150 \times 10}{6} = 250$ km.

Problem [8]

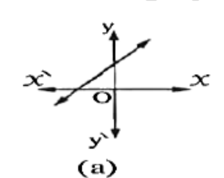
$\because n \propto \frac{1}{x}$ $\quad \therefore \frac{n_1}{n_2} = \frac{x_2}{x_1}$

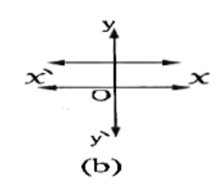
$\therefore \frac{4}{n_2} = \frac{8}{6}$ $\quad \therefore n_2 = \frac{4 \times 6}{8} = 3$ hours.

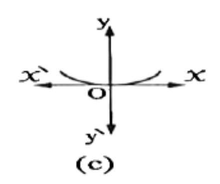
EX. (1): Choose the correct answer:

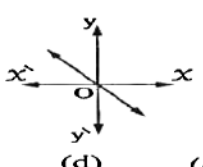
Exercises

1. The graphical form representing the direct variation between x and y is ....


(a)


(b)


(c)


(d)

(El-Sharkia 16)

2. 🕮 The relation which represents a direct variation between the two variables x and y is .... *(El-Menia 19)*

(a) $x y = 5$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$

3. Which of the following relations represents an inverse variation between the two variables x and y ? *(El-Beheira 15)*

(a) $y = x + 5$ (b) $y = 4x$ (c) $\frac{x}{y} = \frac{5}{7}$ (d) $x y = 11$

4. If y varies inversely as x^2 , k is a constant , then ....

(a) $y = k x^2$ (b) $y = k - x^2$ (c) $y = \frac{k}{x^2}$ (d) $y = \frac{k}{x}$

5. If x and y are two variables , $\frac{x_1 y_1}{x_2 y_2} = 1$, then $y \propto$ ....

(a) x (b) $\frac{1}{x}$ (c) x^2 (d) $\frac{1}{x^2}$

6. If $y \propto x$ and y = 5 when $x = 3$, then the constant proportional equals ....

(a) 15 (b) 5 (c) 3 (d) $\frac{5}{3}$

7. 🕮 If y varies inversely with x and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals .... *(El-Beheira 16 – Beni Suef 15 – Alexandria 12)*

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

8. If $x y^5$ = constant , then x varies inversely as .... *(Ismailia 08)*

(a) $\frac{1}{5}$ (b) y^5 (c) y (d) y^2

9. If $y \propto \frac{1}{\sqrt{x}}$, then x varies .... *(Matrouh 09)*

(a) directly as y^2 (b) inversely as y^2 (c) inversely as y (d) inversely as $\sqrt{y}$

10. If $y^2 + 4x^2 = 4xy$, then .... *(South Sinai 19 – Alexandria 15)*

(a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x}$

11. If $x^2 y^2 + \frac{1}{4} = x y$, then ....

(a) $x \propto y$ (b) $y \propto x$ (c) $2x \propto 5y$ (d) $y \propto \frac{1}{x}$

12. If $y = 3x - 6$, then $y \propto$ .... *(El-Sharkia 14)*

(a) x (b) $3x$ (c) $x - 2$ (d) $3x + 6$

13. If $\frac{y+3}{y} = \frac{x+2}{x}$ where $x \neq y \neq$ zero , then $y \propto$ .... *(Ismailia 14)*

(a) x (b) $\frac{1}{x}$ (c) $x + 2$ (d) $x + 5$

14. If $y - x = \frac{2}{x} - \frac{2}{y}$ where $x \neq y \neq 0$, then ....

(a) $y \propto x + 1$ (b) $y \propto x$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

15. 🕮 If the total cost of a trip is (y) , some of it is constant (a) and the other is directly proportional with the number of participants (x) , then .... *(Ismailia 11)*

(a) $y = a x$ (b) $y = \frac{a}{x}$

(c) $y = a + \frac{m}{x}$ (m is constant ≠ 0) (d) $y = a + m x$ (m is a constant ≠ 0)

EX. (2): Answer the following:

1

From the data in the following table , answer the following questions :

1. Show the type of variation between x and y
2. Find the constant of variation.
3. Find the value of y at $x = 3$
4. Find the value of x at $y = 2\frac{2}{5}$

x	2	4	6
y	6	3	2

(Damietta 16 – Ismailia 18) « 12 , 4 , 5 »

2

If $y \propto x$ and $y = 14$ when $x = 42$, **find :** *(El-Monofia 15 – Port Said 18 – South Sinai 19)*

1. The relation between x and y
2. The value of y when $x = 60$

« $y = \frac{1}{3}x$, 20 »

3

If $\frac{21x - y}{7x - z} = \frac{y}{z}$, prove that : $y \propto z$ *(Cairo 15 – El-Kalyoubia 18 – Damietta 19)*

4

If $x^4 y^2 - 14x^2 y + 49 = 0$, prove that : $y \propto \frac{1}{x^2}$ *(Alex. 19)*

5

If $x^2 y^2 - 6xy + 9 = 0$, then prove that : y varies inversely as x

(Damietta 13 – South Sinai 14)

6

If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$, **find :** *(El-Kalyoubia 13 – Alex. 17 – North Sinai 19)*

1. The relation between x and y
2. The value of y when $x = 1.5$

« $xy = 6$, 4 »

7

If $y = a - 9$ and $y \propto \frac{1}{x^2}$ and $a = 18$ when $x = \frac{2}{3}$, find the relation between y and x , then deduce the value of y when $x = 1$ *(Kafr El-Sheikh 18 – Suez 18 – Luxor 19)* « $y = \frac{4}{x^2}$, 4 »

8

If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and h = 27 cm. when r = 10.5 cm.

Find h when r = 15.75 cm. « 12 cm. »



Statistics(lessons 1,2)
Collecting data and Dispersion

Unit 3

Mean : -



Remember that

The mean of a set of values $= \dfrac{\text{The total of values}}{\text{Number of values}}$

For example :

- If the marks of 5 pupils are : 25 , 23 , 21 , 22 , 24
- Then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that :

$23 \times 5 = 25 + 23 + 21 + 22 + 24$

Finding the mean of data from the frequency table with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

1 Determine the centres of sets according to the rule :

The centre of a set $= \dfrac{\text{the lower limit + the upper limit}}{2}$

2 Form the vertical table :

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean $= \dfrac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \dfrac{1700}{50} = 34$ marks.

Median : -



Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

• To find the median of a set of values , we do as follows :

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , (23) , 30 , 42

The median = 23

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , [21 , 23] , 24 , 27

The median $= \frac{21 + 23}{2} = 22$

Mode : -



Remember that

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example :

The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Dispersion of a set of values

It means the divergence or the differences among its values.

- The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great , the dispersion is zero if all the values are equal.

i.e. **The dispersion is a measure that expresses how much the sets are homogeneous.**

Remark

If all values (individuals) are equal then the dispersion (σ) is zero

- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion).

Dispersion measurements

1 The range (the simplest measure of dispersion) :

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example :

- If the values of set A are 60 , 58 , 62 , 61 and 59 $\therefore$ The range = 62 – 58 = 4
- If the values of set B are 72 , 78 , 46 , 65 and 39 $\therefore$ The range = 78 – 39 = 39

So the set B is more divergent than the set A

2 Standard deviation :

First : Calculating the standard deviation of a set of values :

The standard deviation $\sigma = \sqrt{\dfrac{\Sigma (x - \overline{x})^2}{n}}$

Where :

x denotes a value of the values ,

$\overline{x}$ denotes the mean of the values and it is read as x bar ,

n denotes the number of values ,

Σ denotes the summation operation.

Example 1

Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

Solution

1. We find the mean of the values.

$$\overline{x} = \frac{\sum x}{n} = \frac{8+9+7+6+5}{5} = 7$$

2. We form the following table :

x	$x - \overline{x}$	$(x - \overline{x})^2$
8	$8 - 7 = 1$	1
9	$9 - 7 = 2$	4
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
5	$5 - 7 = -2$	4
Total		**10**

3. We calculate the standard deviation by substituting in the law :

The standard deviation $(\sigma) = \sqrt{\frac{\sum (x - \overline{x})^2}{n}}$

$\therefore$ The standard deviation $(\sigma) = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.4$

Second : Calculating the standard deviation of a frequency distribution :

For any frequency distribution :

The standard deviation $\sigma = \sqrt{\frac{\sum (x - \overline{x})^2 k}{\sum k}}$

Where :

x represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

$\sum k$ is the sum of frequences and $\overline{x}$ (the mean) $= \frac{\sum (x \times k)}{\sum k}$

(a) Calculating the standard deviation of a simple frequency distribution :

Example 2

The following table shows the distribution of ages of 20 persons in years :

The age	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the ages.

Solution

(1) We find the mean of the ages ($\overline{x}$) by using the following table :

The age (x)	Number of persons (k)	$x \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	**20**	**460**

The mean ($\overline{x}$) $= \dfrac{\Sigma (x \times k)}{\Sigma k} = \dfrac{460}{20} = 23$ years.

(2) We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
Total	**20**			**360**

(3) We calculate the standard deviation from the law :

Standard deviation (σ) $= \sqrt{\dfrac{\Sigma (x - \overline{x})^2 \times k}{\Sigma k}} = \sqrt{\dfrac{360}{20}} = \sqrt{18} \simeq 4.24$ years.

(b) Calculating the standard deviation of a frequency distribution of sets :

Example (3)

The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

Solution

Remember that

The centre of the set = $\frac{\text{lower limit + upper limit}}{2}$

① We find the mean ($\overline{x}$)

by using the following table :

Sets	Centres of sets (x)	Frequence (k)	$x \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		**100**	**6580**

$\therefore$ The mean ($\overline{x}$) $= \frac{\Sigma (x \times k)}{\Sigma k} = \frac{6580}{100} = 65.8$ pounds.

② We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
40	10	40 – 65.8 = – 25.8	665.64	6656.4
50	14	50 – 65.8 = – 15.8	249.64	3494.96
60	20	60 – 65.8 = – 5.8	33.64	672.8
70	28	70 – 65.8 = 4.2	17.64	493.92
80	20	80 – 65.8 = 14.2	201.64	4032.8
90	8	90 – 65.8 = 24.2	585.64	4685.12
Total	**100**			**20036**

③ We calculate the standard deviation by using the law :

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma (x - \overline{x})^2 \times k}{\Sigma k}} = \sqrt{\frac{20036}{100}} \simeq 14.15$ pounds.

EX. (1): Choose the correct answer:

Exercises

1. The difference between the greatest value and the smallest value in a set of individuals is called .... *(El-Sharkia 18 – Souhag 18 – Port Said 19)*
   (a) the range. (b) the arithmetic mean.
   (c) the median. (d) the standard deviation.

2. The positive square root of the average of squares of deviations of the values from their mean is called .... *(Port Said 18 – Kafr El-Sheikh 18 – El-Fayoum 19)*
   (a) the range. (b) the arithmetic mean.
   (c) the standard deviation. (d) the median.

3. The mean of the values : 7 , 3 , 6 , 9 and 5 equals .... *(Alex. 17 – North Sinai 17 – El-Fayoum 18)*
   (a) 3 (b) 6 (c) 4 (d) 12

4. The range of the set of values : 23 , 22 , 15 , 18 and 17 is .... *(Cairo 15)*
   (a) 8 (b) 18 (c) 19 (d) 23

5. If 67 is the greatest value of a set and if the range equals 27 , then the smallest value of this set equals .... *(El-Menia 16)*
   (a) 67 (b) 40 (c) 27 (d) 94

6. The most repeated value in a set of values represents .... *(Damietta 13 – Luxor 16)*
   (a) the median. (b) the range.
   (c) the mode. (d) the mean.

7. If the mean of numbers : $3k - 3$, $3k - 1$, $2k + 1$, $2k + 3$ and $2k + 5$ is 13 , then $k =$ .... *(Alexandria 11)*
   (a) -5 (b) 10 (c) 5 (d) $\frac{1}{5}$

8. If the range of the values 2 , 7 , a , 6 is 8 where $a > 0$, then $a =$ .... *(El-Sharkia 14)*
   (a) 4 (b) 9 (c) -1 (d) 10

9. Which of the following values of a makes the range of the numbers : 53 , a , 58 , 57 , 60 and 55 equal to 9 ? *(El-Dakahlia 16)*
   (a) 63 (b) 61 (c) 51 (d) 50

10. $\frac{\text{Sum of values}}{\text{Number of these values}} =$ .... *(Aswan 13 – Suez 19)*
    (a) range (b) standard deviation (c) mean (d) mode

11. If $2x + 2y = 10$, $x , y \in \mathbb{R}^+$, then the arithmetic mean of the values x and y is .... *(Suez 16)*
    (a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 5 (d) 2

12. The set which has more disperion of the following sets is the set .... *(El-Kalyoubia 15)*
    (a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43
    (c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27

13. The commonest measure of dispersion and the most accurate is the .... *(Damietta 14 – El-Menia 18)*
    (a) range. (b) mean. (c) standard deviation. (d) median.

14. If all individuals are equal in values , then .... *(Southern Sinai 17)*
    (a) $x - \overline{x} > 0$ (b) $x - \overline{x} < 0$ (c) $\sigma = 0$ (d) $\overline{x} = 0$

15. If $\Sigma (x - \overline{x})^2 = 48$ of a set of values and the number of these values = 12 , then $\sigma =$ .... *(Cairo 17 – El-Monofia 19)*
    (a) -4 (b) -2 (c) 2 (d) 4

EX. (2): Answer the following:

1

Calculate the standard deviation for the next data :

[1] 16 , 32 , 5 , 20 , 27 *(El-Sharkia 16 – El-Monofia 17 – El-Gharbia 18 – El-Monofia 19)* « 9.3 »

[2] 72 , 53 , 61 , 70 , 59 *(Luxor 19)* « 7.1 »

2

Calculate the mean and standard deviation of the following data :

[1] 73 , 54 , 62 , 71 , 60 *(Assiut 17)* « 64 , 7.07 »

[2] 13 , 14 , 17 , 19 , 22 (to the nearest 3 decimals digits) *(El-Sharkia 17)* « 17 , 3.286 »

3

The following frequency distribution shows the number of children of some families in a new city : *(Alexandria 15 – El-Beheira 16 – Alex. 19)*

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children. « 2 , 1 »

4

The following are the frequency distribution for a number of defective units found in 100 boxes of manufactured units : *(El-Beheira 14 – El-Beheira 17 – Souhag 18)*

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units. « 1.4 »

5

The following frequency distribution shows the ages of 10 children :

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

(Alex. 17 – Cairo 18 – Giza 18 – Qena 19) « 1.7 years »

6

Calculate the mean and the standard deviation for the following frequency distribution : *(Qena 16 – El-Gharbia 17)*

Set	0 –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	25

« 11.6 , 5.7 »

7

The following distribution table shows the amount of gasoline that a set of cars consumes :

Number of kilometres per litre	5 –	7 –	9 –	11 –	13 –	15 –	Total
Number of cars	3	6	10	12	5	4	40

Find the standard deviation of the number of kilometres per litre. « 2.7 »

Simplest Maths

Maths-Prep3/first term (2024)

Unit 4

1. The main trigonometrical ratios of the acute angle.
2. The main trigonometrical ratios of some angles.

Unit 5

1. Distance between two points.
2. The two coordinates of the midpoint of a line segment.
3. The slope of the straight line.
4. The equation of the straight line given its slope and the intercepted part of y-axis.

Mr.Mohamed El-Shourbagy / 01093149109



Lesson (1)

The main trigonometrical ratios of the acute angle

Unit 4

The relation between each of the degrees, the minutes and the seconds

- The degree = 60 minutes.
- The minute = 60 seconds

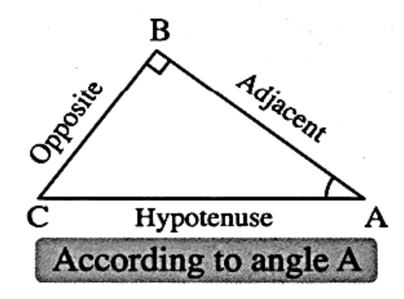
i.e. The degree = $60 \times 60 = 3600$ seconds.

The main trigonometrical ratios of the acute angle

The trigonometrical ratio of the acute angle

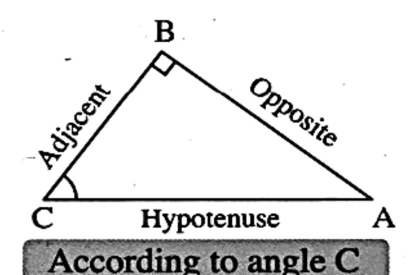
It is the ratio between two side lengths of the right-angled triangle that contains this angle.

i.e. If Δ ABC is a right-angled triangle at B , then :



According to angle A

1. $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
2. $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
3. $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$



According to angle C

1. $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
2. $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
3. $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$.

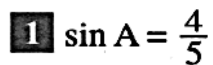
For example

In the opposite figure :

If Δ ABC is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :

A
5 cm.
3 cm.
C
4 cm.
B



1. $\sin A = \frac{4}{5}$
2. $\cos A = \frac{3}{5}$
3. $\tan A = \frac{4}{3}$

1. $\sin C = \frac{3}{5}$
2. $\cos C = \frac{4}{5}$
3. $\tan C = \frac{3}{4}$

We can deduce that :

The sine of any angle equals the cosine of its complementary and vice versa

i.e. If $\angle A$ and $\angle B$ are acute angles , and $\sin A = \cos B$

then : $m(\angle A) + m(\angle B) = 90°$

Generally

The tangent of the angle $= \dfrac{\text{The sine of the angle}}{\text{The cosine of the angle}}$

Examples : Solved Problems

1 If : $\tan x = \frac{1}{\sqrt{3}}$, x is an acute angle. **Find :** $\sin x \tan \frac{3x}{2} + \cos 2x$

2 XYZ is a right-angled triangle at Y , XY = 3 cm. , XZ = 5 cm. , **then find the value of :**

(1) $\tan X \times \tan Z$ (2) $\sin^2 X + \sin^2 Z$

3 ABC is a right-angled triangle at B and $\sin A = 0.6$ *(Kafr El-Sheikh 2013)*

Find the value of : $\sin A \cos C + \cos A \sin C$ « 1 »

4 ABCD is a trapezoid in which : $\overleftrightarrow{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$, if AB = 3 cm.

, AD = 6 cm. and BC = 10 cm.

Prove that : $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$ *(El-Kalyoubia 2013)*

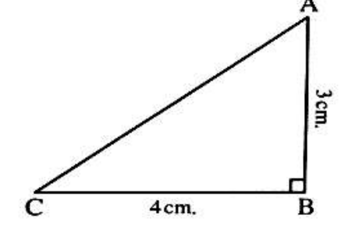
5 **In the opposite figure :**

Δ ABC where $m(\angle B) = 90°$,

AB = 3 cm. , BC = 4 cm.

Find : (1) $\sin^2 A + \sin^2 C$

(2) $\tan A \times \tan C$



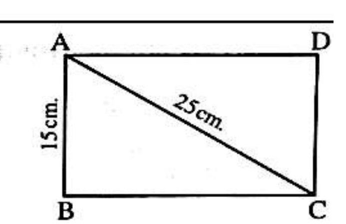
6 **In the opposite figure :**

ABCD is a rectangle in which

AB = 15 cm. and AC = 25 cm.

Find : (1) $m(\angle ACB)$

(2) The surface area of the rectangle ABCD



Solution

1

$\because \tan X = \frac{1}{\sqrt{3}} \quad \therefore X = 30^\circ$

$\therefore \sin X \tan \frac{3X}{2} + \cos 2X = \sin 30^\circ \tan 45^\circ + \cos 60^\circ$

$= \frac{1}{2} \times 1 + \frac{1}{2} = 1$

2

$\because m(\angle Y) = 90^\circ$

$\therefore (YZ)^2 = (5)^2 - (3)^2 = 16$

$\therefore YZ = 4$ cm.

(1) $\therefore \tan X \times \tan Z = \frac{4}{3} \times \frac{3}{4} = 1$

(2) $\sin^2 X + \sin^2 Z = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = 1$

3

$\because \sin C = \frac{6}{10} = \frac{3}{5}$. Assuming that

AB = 3 length unit

$\therefore$ AC = 5 length unit

$\therefore$ BC = 4 length unit

$\therefore \sin A \cos C + \cos A \sin C$

$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{16}{25} + \frac{9}{25} = 1$

4

Draw $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$\therefore$ BF = AD = 6 cm.

$\therefore$ FC = 4 cm. , DF = AB = 3 cm.

$\therefore$ from Δ DFC

which is right-angled at F

$(DC)^2 = 3^2 + 4^2 = 25$

$\therefore$ DC = 5 cm.

$\cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

5

$\because m(\angle B) = 90^\circ \quad \therefore (AC)^2 = (4)^2 + (3)^2 = 25$

$\therefore AC = \sqrt{25} = 5$ cm.

$\therefore \sin^2 A + \sin^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1$

, $\tan A \times \tan C = \frac{4}{3} \times \frac{3}{4} = 1$

6

In Δ ABC : $\because m(\angle B) = 90^\circ$ (Properties of rectangle)

$\therefore \sin(\angle ACB) = \frac{15}{25} \quad \therefore m(\angle ACB) \simeq 36^\circ\, 52'\, 12''$

, $(BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore BC = \sqrt{400} = 20$ cm.

$\therefore$ The surface area of the rectangle ABCD

$= 15 \times 20 = 300 \text{ cm}^2$.

EX. (1): Choose the correct answer:

Exercises

1. For any acute angle A , tan A = ... *(Ismailia 12)*

   (a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

2. If X , y are measures of two complementary angles and $\sin X = \frac{3}{5}$, then cos y = ... *(Giza 17 – El-Beheira 18)*

   (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

3. For any two acute angles A and B if sin A = cos B , then m (∠ A) + m (∠ B) = ...

   (a) 30° (b) 60° (c) 90° (d) 180°

4. If sin 70° = cos X where X is an acute angle , then X = ... *(El-Kalyoubia 18)*

   (a) 60° (b) 45° (c) 10° (d) 20°

5. In Δ ABC , if m (∠ A) = 85° and sin B = cos B , then m (∠ C) = ...

   (El-Beheira 17 – El-Dakahlia 19)

   (a) 30° (b) 45° (c) 50° (d) 60°

6. In Δ ABC : if m (∠ B) = 90° , then sin A + cos C = ... *(El-Monofia 17)*

   (a) 2 sin A (b) 2 sin C (c) 2 sin B (d) 2 cos A

7. Δ ABC is a right-angled triangle at A , then cosine angle B : sine angle C equals ....

   (El-Sharkia 18)

   (a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

8. DEF is a right-angled triangle at E , which of the following relations is false ?

   (a) tan D × tan F = 1 (b) sin D = cos F (c) cos D = sin F (d) cos D = sin E

   (El-Dakahlia 16)

9. **In the opposite figure :**

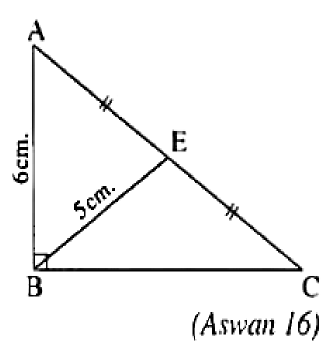
   Δ ABC is a right-angled triangle at B

   , $\overline{BE}$ is a median , BE = 5 cm.

   , AB = 6 cm. , then sin C = ...

   (a) $\frac{5}{6}$ (b) $\frac{3}{5}$

   (c) $\frac{6}{5}$ (d) $\frac{5}{3}$



(Aswan 16)

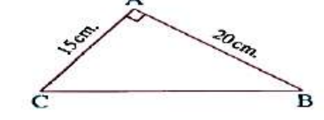
EX. (2): Answer the following:

1 If the ratio between the measures of two supplementary angles is 3 : 5 , find the measure of each one by degree measure. *(El-Beheira 14 – Aswan 15 – El-Gharbia 19)* « $67° 30'$, $112° 30'$ »

2 If the ratio between the measures of two complementary angles is 3 : 4 , find the degree measure of the greater angle in measure. « $51° 25' 43''$ »

3 If the ratio between the measures of the interior angles of the triangle is 3 : 4 : 7 , find the degree measure of each angle. *(El-Beheira 13)* « $38° 34' 17''$, $51° 25' 43''$, $90°$ »

4 **In the opposite figure :**



ABC is a triangle in which : $m(\angle A) = 90°$, AC = 15 cm. and AB = 20 cm.

Prove that : $\cos C \cos B - \sin C \sin B = \text{zero}$

(El-Kalyoubia 16 – El-Beheira 17 – El-Kalyoubia 18 – El-Menia 19)

5 XYZ is a right-angled triangle at Z where : XZ = 7 cm. and XY = 25 cm.

Find the value of each of the following :

1. $\tan X \times \tan Y$
2. $\sin^2 X + \sin^2 Y$

(Port Said 18) « 1 , 1 »

6 ABC is a right-angled triangle at B , if $2\,AB = \sqrt{3}\,AC$

Find the main trigonometrical of the angle C

(Alexandria 15 – El-Dakahlia 18 – Aswan 19) « $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$, $\sqrt{3}$ »

7 ABCD is an isosceles trapezoid in which : $\overline{AD} \parallel \overline{BC}$, AD = 4 cm. , AB = 5 cm. and BC = 12 cm.

Prove that : $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$ *(New Valley 17)*

8 ABCD is a trapezoid in which : $\overrightarrow{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$ if AB = 3 cm. , AD = 6 cm. and BC = 10 cm.

Prove that : $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$ *(El-Kalyoubia 13 – El-Monofia 17 – Matrouh 18)*

9 ABC is a right-angled triangle at B , if AB : AC = 3 : 5 *(Aswan 2013)*

Find : the main trigonometrical of $\angle A$ « $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ »

10 ABC is an isosceles triangle in which : AB = AC and $\sin \frac{A}{2} = \frac{4}{5}$ *(Red Sea 2013)*

Find cos B without using the calculator. « $\frac{4}{5}$ »



Lesson (2)

The main trigonometrical ratios of some angles

The trigonometrically ratios of the two angels 30°,60°

The trigonometrically ratios of the angels 45°

sn.	Angle	30°	60°	45°
1	Sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
2	Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
3	Tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Using the calculator

First : Finding the trigonometrical ratios of a given angle :

In the calculator, there are three keys : [sin] , [cos] , [tan]

1 The key [sin] means sine (sin)

2 The key [cos] means cosine (cos)

3 The key [tan] means tangent (tan)

Using these keys gives us the main trigonometrical ratios of any angle if its measure is known.

Second : Finding the measure of the angle if one of its trigonometrical ratios is given :

Find A in each of the following, where A is the measure of an acute angle :

1 $\sin A = 0.8$ **2** $\cos A = 0.7152$ **3** $\tan A = 1.5156$

1 Use the keys of the calculator as the following sequence from left :

[shift] [sin] [·] [8] [=] [°,,,] $\therefore A \simeq 53^\circ\ 7'\ 48''$

2 Use the keys of the calculator as the following sequence from left :

[shift] [cos] [·] [7] [1] [5] [2] [=] [°,,,] $\therefore A \simeq 44^\circ\ 20'\ 25''$

3 Use the keys of the calculator as the following sequence from left :

[shift] [tan] [1] [·] [5] [1] [5] [6] [=] [°,,,] $\therefore A \simeq 56^\circ\ 34'\ 59''$

Examples : Solved problems

1	**Find the value of :** $\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$ **2014 Exam (13) Question (2) (a)**
2	**Without using the calculator prove that :** $\sin^2 30° = 5 \cos^2 60° - \tan^2 45°$ **2014 Exam (4) Question (3) (a)**
3	**Without using the calculator :** (1) **Find the value of :** x , if $3x = 2 \sin 30° \tan^2 45°$ (2) **Prove that :** $2 \cos^2 30° = \cos 60° + 1$ **2014 Exam (15) Question (2) (a)**
4	**Without calculator and showing the steps of solution :** **Calculate value of :** $x°$ such that $x°$ is acute angle and satisfies that : $\sqrt{2} \sin x = \sin 30° \cos 60° + \cos 30° \sin 60°$ **2014 Exam (10) Question (2) (b)**
5	**Find the value of :** x where $0° < x < 90°$ given that : $\tan x = \dfrac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°}$ **2014 Exam (8) Question (4) (b)**
6	**Without using the calculator :** If : $2 \tan x = \tan^2 60° - 2 \sin 30°$ **Find the value of :** x (Where x is acute angle) **2014 Exam (12) Question (3) (b)**

Solution

1	$\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$ $= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{2} + \frac{3}{4}$ $= -\frac{1}{2}$	2	L.H.S. $= \sin^2 30° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (1) $\therefore$ R.H.S. $= 5 \cos^2 60° - \tan^2 45°$ $= 5\left(\frac{1}{2}\right)^2 - (1)^2 = \frac{5}{4} - 1 = \frac{1}{4}$ (2) From (1) and (2) : $\therefore \sin^2 30° = 5 \cos^2 60° - \tan^2 45°$
3	(1) $3x = 2 \sin 30° \tan^2 45° = 2 \times \frac{1}{2} \times (1)^2 = 1$ $\therefore x = \frac{1}{3}$	4	$\because \sqrt{2} \sin x = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$ $\therefore \sqrt{2} \sin x = 1$ $\therefore \sin x = \frac{1}{\sqrt{2}}$ $\therefore x = 45°$
5	$\because \tan x = \dfrac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°}$ $\therefore \tan x = \dfrac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}} = 1$ $\therefore x = 45°$	6	$\because 2 \tan x = \tan^2 60° - 2 \sin 30°$ $\therefore 2 \tan x = \left(\sqrt{3}\right)^2 - 2 \times \frac{1}{2}$ $\therefore 2 \tan x = 2$ $\therefore \tan x = 1$ $\therefore x = 45°$

EX. (1): Choose the correct answer:

Exercises

1 If $\cos X = \frac{1}{2}$ where X is an acute angle, then $m(\angle X) = \dots\dots$ *(Cairo 13)*

(a) 90° (b) 60° (c) 45° (d) 30°

2 If $\sin X = \frac{1}{2}$ where X is an acute angle, then $m(\angle X) = \dots\dots$ *(North Sinai 17 – Alex. 18)*

(a) 90° (b) 60° (c) 45° (d) 30°

3 If $\tan X = \frac{1}{\sqrt{3}}$ where X is an acute angle, then $\tan 2X = \dots\dots$

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) 3

4 If $\cos X = \frac{\sqrt{3}}{2}$ where X is a measure of an acute angle, then $\sin 2X = \dots\dots$

(El-Gharbia 18 – Red Sea 19)

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

5 If $2\sin X = \tan 60°$ where X is an acute angle, then $m(\angle X) = \dots\dots$ *(Souhag 11)*

(a) 30° (b) 45° (c) 60° (d) 40°

6 If $\tan 3X = \sqrt{3}$ where $3X$ is an acute angle, then $m(\angle X) = \dots\dots$ *(Ismailia 15)*

(a) 20° (b) 30° (c) 45° (d) 60°

7 If $\sin 2X = \frac{\sqrt{3}}{2}$, then $X = \dots\dots$ (where $2X$ is an acute angle) *(Giza 11)*

(a) 20° (b) 30° (c) 45° (d) 60°

8 If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is an acute angle , then $m(\angle X) =$ ....

(a) 30° (b) 45° (c) 60° (d) 120°

9 If $\cos(X + 10°) = \frac{1}{2}$ where $(X + 10°)$ is an acute angle , then $X =$ .... *(El-Fayoum 11)*

(a) 30° (b) 40° (c) 50° (d) 70°

10 If $\tan(2X - 5°) = 1$ where X is an acute angle , then $X =$ ....° *(El-Gharbia 16)*

(a) 45 (b) 35 (c) 25 (d) 15

11 If $\sin(X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is the measure of an acute angle

, then $\tan(X + 20°) =$ .... *(El-Dakahlia 11)*

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

12 If X and y are complementary angles where $X : y = 1 : 2$, then $\sin X + \cos y =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

(El-Beheira 15)

13 In the triangle ABC if : $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 5$, then $\cos B =$ ....

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

(El-Gharbia 16)

14 The tangent of an acute angle of the right isosceles triangle is equal to ....

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{2}}{2}$

(El-Dakahlia 16)

15 Δ ABC is right-angled at A , if $\tan B = 1$, then $\tan C - \sin C \cos C =$ ....

(a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$

(Red Sea 16)

16 If the straight line : $y = X \sin 30° + c$ passes through the point (4 , 6) , then c = ...

(a) 4 (b) 6 (c) 8 (d) 2

(El-Monofia 16)

EX. (2): Answer the following:

1 **Without using the calculator , prove each of the following :**

1. $\sin 60° = 2 \sin 30° \cos 30°$ *(Giza 14 – Souhag 18 – Giza 19)*
2. $\cos 60° = 2 \cos^2 30° - 1$ *(Alex. 14 – Port Said 18 – North Sinai 19)*
3. $2 \cos^2 30° - 1 = 1 - 2 \sin^2 30°$ *(El-Sharkia 15)*
4. $\cos 60° = \cos^2 30° - \sin^2 30°$ *(Alex. 17 – El-Fayoum 18 – South Sinai 19)*
5. $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$ *(Port Said 14 – Matrouh 17 – Damietta 19)*

2 **Find the value of X :**

1. $X \sin^2 45° = \tan^2 60°$ *(Souhag 17)* « 6 »
2. $X \sin 30° \cos^2 45° = \sin^2 60°$ *(Alex. 15 – South Sinai 16 – Alex. 19)* « 3 »
3. $X \sin 45° \cos 45° \tan 60° = \tan^2 45° - \cos^2 60°$ « $\frac{\sqrt{3}}{2}$ »
4. $4 X = \cos^2 30° \tan^2 30° \tan^2 45°$ *(Alex. 17 – El-Fayoum 19)* « $\frac{1}{16}$ »

3 **Find the value of X in each of the following :**

1. $\tan X = 4 \sin 30° \cos 60°$ where X is an acute angle. *(North Sinai 17 – Damietta 18 – El-Gharbia 19)* « 45° »
2. $\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$ where $0° < X < 90°$ *(Cairo 17)* « 30° »
3. $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$ where X is an acute angle. *(Giza 18)* « 30° »

4 ABC is an isosceles triangle in which AB = AC = 12.6 cm. and m (∠ C) = 84° 24'

Find the length of $\overline{BC}$ to the nearest one decimal number. « 2.5 cm. »

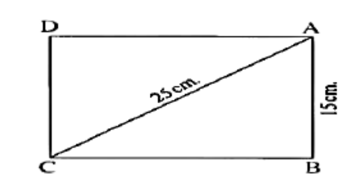
5 **In the opposite figure :**

ABCD is a rectangle in which :

AB = 15 cm. and AC = 25 cm.

Find :

1. m (∠ ACB)
2. The area of the rectangle ABCD *(Alex. 16 – Qena 17 – El-Kalyoubia 19)* « 36° 52' 12" , 300 cm.² »



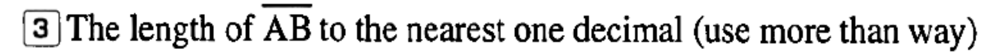
6

In the opposite figure :

ABCD is a parallelogram of surface area = 96 cm^2

BE : EC = 1 : 3 , $\overline{AE} \perp \overline{BC}$ and AE = 8 cm.

Find : [1] The length of $\overline{AD}$ [2] m (∠ B)



[3] The length of $\overline{AB}$ to the nearest one decimal (use more than way)

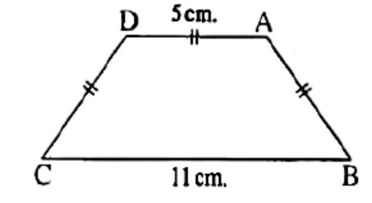
« 12 cm. , 69° 26′ 38″ , 8.5 cm. »

7

In the opposite figure :

ABCD is an isosceles trapezium in which :

AB = AD = DC = 5 cm., BC = 11 cm. **Find :**



[1] m (∠ B) , m (∠ A)

[2] The area of the trapezium ABCD

(Matrouh 13) « 53° 7′ 48″ , 126° 52′ 12″ , 32 cm^2 »

Explaining

Lessons (1,2)

Unit 5

Distance between two points

The two coordinates of the midpoint of a line segment

i.e. The distance between the two points M and N equals $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

and we know that :

$(x_2-x_1)^2=(x_1-x_2)^2$, and similarly : $(y_2-y_1)^2=(y_1-y_2)^2$, therefore :

The distance between the two points M and N equals also $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Generally : The distance between two points =

$\sqrt{\text{square of the difference between } x\text{–coordinates} + \text{square of the difference between } y\text{–coordinates}}$

For example :

- The distance between the two points M (3 , 6) and N (– 1 , 4) is :

$$MN=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(-1-3)^2+(4-6)^2}=\sqrt{(-4)^2+(-2)^2}$$

$$=\sqrt{16+4}=\sqrt{20}=2\sqrt{5} \text{ length unit.}$$

Remark 1

To prove that three given points are collinear (*i.e.* they lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

Remark 2

- To prove that the points : A , B and C are vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)

  We compare between $(AC)^2$ and $(AB)^2+(BC)^2$ as the following :

  1. If $(AC)^2>(AB)^2+(BC)^2$, then the triangle is obtuse-angled at B
  2. If $(AC)^2=(AB)^2+(BC)^2$, then the triangle is right-angled at B
  3. If $(AC)^2<(AB)^2+(BC)^2$, then the triangle is acute-angled.

Remark 3

If ABCD is a quadrilateral :

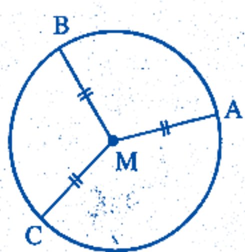
1. To prove that ABCD is a parallelogram , we prove that : AB = CD , BC = AD
2. To prove that ABCD is a rhombus , we prove that : AB = BC = CD = DA

3 To prove that ABCD is a rectangle , we prove that : AB = CD , BC = AD , AC = BD

4 To prove that ABCD is a square , we prove that : AB = BC = CD = DA , AC = BD

Remark 4

- To prove that : Three points as A , B and C lie on a same circle of centre M
  we prove that : MA = MB = MC
- If A $\in$ the circle M , then the radius length of this circle (r) = MA
- Remember that :
  - Circumference of the circle = $2 \pi r$
  - Area of the circle = πr^2



Lesson [2] : Two Coordinates Of Midpoint Of A Line Segment

First point : A (X_1 , y_1) , **Second point : A (X_2 , y_2)**

Midpoint point: M (m_x , m_y) then

$$M(m_x, m_y) = \left(\frac{X_1 + X_2}{2}, \frac{y_1 + y_2}{2}\right),$$

$X_1 = m_x \times 2 - X_2$ $y_1 = m_y \times 2 - y_2$

For Example : -

- If A (1 , 5) , B (3 , 1) and M is the midpoint of $\overline{AB}$, then :

$$M = \left(\frac{1+3}{2}, \frac{5+1}{2}\right) = (2, 3)$$

- If X (3 , – 2) , Y (– 1 , – 4) and M is the midpoint of $\overline{XY}$, then :

$$M = \left(\frac{3+(-1)}{2}, \frac{-2+(-4)}{2}\right) = (1, -3)$$

Remark : -

If $\overline{AB}$ is a diameter in a circle of centre M , then M is the midpoint of $\overline{AB}$

Examples

1 🕮 Prove that the triangle with vertices of points : A (5 , – 5) , B (– 1 , 7) and C (15 , 15) is a right-angled triangle at B , then calculate its area.

(Beni Suef 2013 – El-Monofia 2014) « 120 square unit »

2 **In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :**

A (–1 , 1) , B (0 , 5) , C (5 , 6) and D (4 , 2) *(Suez 2011)*

3 **Prove that :** The points A (0 , 1) , B (4 , 5) , C (1 , 8) and D (– 3 , 4) are vertices of a rectangle and find its diagonal length. *(Souhag 2009)* « $5\sqrt{2}$ length units»

4 **Prove that :** The points A (3 , – 1) , B (– 4 , 6) and C (2 , – 2) lie on the same circle whose centre is M (– 1 , 2) , then find the circumference of the circle where $\pi = 3.14$

(Alex. 2015 – Cairo 2015 – El-Sharkia 2013) « 31.4 length units »

5 🕮 If A (x , 3) , B (3 , 2) and C (5 , 1) and AB = BC , then find the value of x

(El-Beheira 2015 – Port Said 2014) « 5 or 1 »

6 If C (4 , 6) is the midpoint of $\overline{AB}$ where A (x , 3) and B (6 , y) , then find the value of each of : x and y *(Cairo 2015)* « 2 , 9 »

7 🕮 **If C is the midpoint of $\overline{AB}$, then find x , y in each of the following cases :**

A (x , 3) , B (6 , y) , C (4 , 6) *(Luxor 2013)* « 2 , 9 »

8 $\overline{AB}$ is a diameter in a circle M , if B (8 , 11) and M (5 , 7) **Find :**

(1) The coordinates of A

(2) The perimeter of the circle. where ($\pi = 3.14$) *(Assiut 2014)* « A (2 , 3) , 31.4 length unit »

9 🕮 If the points A (3 , 2) , B (4 , –3) , C (–1 , –2) and D (–2 , 3) are vertices of the rhombus. **Find :**

(1) The coordinates of the point of intersection of the two diagonals.

(2) The area of the rhombus ABCD *(Suez 2014)* « (1 , 0) , 24 square unit »

The solutions

Problem [1]

$AB = \sqrt{(-1-5)^2 + (7+5)^2} = \sqrt{36+144}$
$= \sqrt{180}$ length unit

, $BC = \sqrt{(15+1)^2 + (15-7)^2} = \sqrt{256+64}$
$= \sqrt{320}$ length unit

and $CA = \sqrt{(5-15)^2 + (-5-15)^2} = \sqrt{100+400}$
$= \sqrt{500}$ length unit

, $\because (CA)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is right angled at B

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$
$= \frac{1}{2} \times \sqrt{180} \times \sqrt{320}$
$= 120$ square units

Problem [2]

$AB = \sqrt{(0+1)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $BC = \sqrt{(5-0)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

, $CD = \sqrt{(4-5)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $DA = \sqrt{(-1-4)^2 + (1-2)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

Problem [3]

$\because AB = \sqrt{(0-4)^2 + (1-5)^2} = \sqrt{16+16}$
$= \sqrt{32} = 4\sqrt{2}$ length unit.

, $BC = \sqrt{(4-1)^2 + (5-8)^2} = \sqrt{9+9}$
$= \sqrt{18} = 3\sqrt{2}$ length unit.

, $CD = \sqrt{(1+3)^2 + (8-4)^2} = \sqrt{16+16}$
$= \sqrt{32} = 4\sqrt{2}$ length unit.

, $AD = \sqrt{(0+3)^2 + (1-4)^2} = \sqrt{9+9}$
$= \sqrt{18} = 3\sqrt{2}$ length unit.

$\because AB = CD$, $BC = AD$

$\therefore$ the figure ABCD is a parallelogram

, $\because AC = \sqrt{(0-1)^2 + (1-8)^2} = \sqrt{1+49}$
$= \sqrt{50} = 5\sqrt{2}$ length unit.

, $BD = \sqrt{(4+3)^2 + (5-4)^2} = \sqrt{49+1}$
$= \sqrt{50} = 5\sqrt{2}$ length unit.

$\therefore AC = BD = 5\sqrt{2}$ length unit.

$\therefore$ The figure ABCD is a rectangle its diagonal length $= 5\sqrt{2}$ length unit.

Problem [4]

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$
$= \sqrt{25} = 5$ length unit

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$
$= \sqrt{25} = 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$
$= \sqrt{25} = 5$ length unit

$\therefore MA = MB = MC$

$\therefore$ A , B and C lie on the circle M which its radius length is 5 length units

$\therefore$ circumference of the circle $= 2\pi r$
$= 2 \times 3.14 \times 5$
$= 31.4$ length units

Problem [5]

$BC = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$ length unit.

$\therefore AB = \sqrt{5}$ *length unit.*

$\therefore \sqrt{(x-3)^2 + (3-2)^2} = \sqrt{5}$ "squaring the two sides"

$\therefore (x-3)^2 + (1)^2 = 5$ $\qquad \therefore x^2 - 6x + 9 + 1 = 5$

$\therefore x^2 - 6x + 5 = 0$ $\qquad \therefore (x-5)(x-1) = 0$

$\therefore x = 5$ or $x = 1$

Problem [6]

$\because$ C is the midpoint of $\overline{AB}$

$\because (4, 6) = \left(\frac{x+6}{2}, \frac{y+3}{2}\right)$

$\therefore \frac{x+6}{2} = 3$ $\qquad \therefore x + 6 = 8$ $\qquad \therefore x = 2$

, $\frac{y+3}{2} = 6$ $\qquad \therefore y + 3 = 12$ $\qquad \therefore y = 9$

Problem [7]

$\because (4, 6) = \left(\frac{x+6}{2}, \frac{3+y}{2}\right)$

$\therefore \frac{x+6}{2} = 4$ $\qquad \therefore x + 6 = 8$ $\qquad \therefore x = 2$

$\frac{3+y}{2} = 6$ $\qquad \therefore 3 + y = 12$ $\qquad \therefore y = 9$

Problem [8]

Let : A (x, y)

$\therefore (5, 7) = \left(\frac{x+8}{2}, \frac{y+11}{2}\right)$

$\therefore \frac{x+8}{2} = 5$ $\qquad \therefore x + 8 = 10$

$\therefore x = 2$, $\frac{y+11}{2} = 7$ $\qquad \therefore y + 11 = 14$

$\therefore y = 3$ $\qquad \therefore A(2, 3)$

$\therefore r = MA = \sqrt{(5-2)^2 + (7-3)^2}$
$= \sqrt{9+16} = 5$ length unit.

$\therefore$ The circumference of the circle $= 2\pi r$
$= 2 \times 3.14 \times 5 = 31.4$ length unit

Problem [9]

(1) Let E be the point of intersection of the two diagonals

$\therefore$ The coordinates of $E = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

(2) $AC = \sqrt{(-1-3)^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32}$
$= 4\sqrt{2}$ length units

$BD = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

The area of the rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$
$= 24$ square unit

EX. (1): Choose the correct answer (A):

Exercises

1. The distance between the two points $(3, a)$ and $(-1, a)$ is .... length unit. *(Ismailia 17)*
   (a) 16 (b) 9 (c) 5 (d) 4

2. The distance between the point $(\sqrt{3}, 1)$ and the point of origin is .... *(Souhag 18)*
   (a) 4 (b) 3 (c) 2 (d) 1

3. If the distance between the two points $(a, 0)$, $(0, 1)$ is unit length, then $a =$ ....
   (a) 1 (b) 0 (c) -1 (d) $\sqrt{2}$

4. The radius length of the circle whose centre is $(7, 4)$ and passes through $(3, 1)$ equals .... length units.
   (a) 5 (b) -5 (c) 2.5 (d) 25

5. If ABCD is a square and A $(3, 5)$ and B $(4, 2)$, then the area of the square ABCD equals .... area unit.
   (a) $\sqrt{10}$ (b) 10 (c) $4\sqrt{10}$ (d) 40

6. If ABCD is a rhombus and A $(-1, 7)$, B $(-3, 1)$, then the perimeter of the rhombus ABCD = ... length unit.
   (a) 40 (b) $4\sqrt{52}$ (c) $8\sqrt{10}$ (d) $2\sqrt{10}$

7. In the Cartesian coordinates plane, a point that is at a distance 2 length unit from the origin may be .... *(Cairo 09)*
   (a) $(1, 2)$ (b) $(2, 1)$ (c) $(0, 2)$ (d) $(-3, 5)$

8. The distance between the point $(-5, -2)$ and y-axis is .... length unit. *(El-Gharbia 16)*
   (a) -5 (b) -2 (c) 2 (d) 5

9. The distance between the point $(5, \tan^2 60°)$ and the x-axis is .... length unit. *(Suez 17)*
   (a) 5 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$

10. The distance between the point $(\ell, -4)$ and y-axis is .... length unit, where $\ell \in \mathbb{R}$ *(Damietta 18)*
    (a) 4 (b) ℓ (c) -4 (d) $|\ell|$

11. The perpendicular distance between the two straight lines, $y - 3 = 0$, $y + 2 = 0$ equals .... length units. *(Alex. 17 – El-Fayoum 17)*
    (a) 5 (b) 1 (c) 2 (d) 3

12. A circle its centre is the origin and its radius length is 2 length unit, which of the following points belongs to the circle? *(El-Gharbia 14 – Beni Suef 16 – El-Beheira 17)*
    (a) $(1, 2)$ (b) $(-2, 1)$ (c) $(\sqrt{3}, 1)$ (d) $(\sqrt{2}, 1)$

EX. (1): Choose the correct answer (B):

1. If $(4, -3)$ is the midpoint of $\overline{XY}$ where $X(5, -2)$, then Y is .... *(El-Menia 16)*

   (a) $(4.5, 4.5)$ (b) $(4, 3)$ (c) $(3, 4)$ (d) $(3, -4)$

2. If $C(-3, y)$ is the midpoint of $\overline{AB}$ where $A(x, -6)$ and $B(1, -8)$ *(Qena 18)*

   , then $x + y =$ ....

   (a) -11 (b) 11 (c) -18 (d) -14

3. If $\overline{AB}$ is a diameter in a circle where $A(3, -5)$ and $B(5, 1)$

   , then the centre of the circle is .... *(El-Fayoum 18 – Matrouh 19)*

   (a) $(4, -2)$ (b) $(4, 2)$ (c) $(2, -2)$ (d) $(8, -2)$

4. If ABCD is a square where $A(3, 4)$, $C(5, 6)$, then the midpoint of its diagonal is .... *(El-Menia 18)*

   (a) $(8, 10)$ (b) $(10, 8)$ (c) $(4, 5)$ (d) $(15, 24)$

5. If $M(1, 2)$ is the intersection point of the two diagonals of the parallelogram ABCD where $A(2, 5)$, then C is ....

   (a) $(0, 2)$ (b) $(0, -1)$ (c) $(-4, 1)$ (d) $(-1, 0)$

6. If $\left(\frac{1}{2}, \frac{5}{2}\right)$ is the midpoint of $\overline{AB}$ where $A(1, -1)$ and $B(x, 6)$, then $x =$ ....

   (a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

7. If the x-axis bisects $\overline{AB}$ such that $A(3, 2)$ and $B(-2, y)$, then $y =$ .... *(El-Dakahlia 17)*

   (a) 3 (b) 2 (c) -2 (d) 4

EX. (2): Answer the following:

1	Prove that the triangle with vertices of points : $A(5, -5)$, $B(-1, 7)$ and $C(15, 15)$ is a right-angled triangle at B, then calculate its area. *(Beni Suef 2013 – El-Monofia 2014)* « 120 square unit »
2	If $A(-1, -1)$, $B(2, 3)$ and $C(6, 0)$ **Prove that :** ΔABC is a right-angled triangle, then find its area. *(Alexandria – Beni Suef 2011)* « 12.5 square units »
3	**In each of the following, prove that the points A, B, C and D are vertices of a parallelogram where :** $A(-1, 1)$, $B(0, 5)$, $C(5, 6)$ and $D(4, 2)$ *(Suez 2011)*
4	**In each of the following, prove that the points A, B, C and D are vertices of a parallelogram where :** $A(-2, 4)$, $B(5, -3)$, $C(7, 1)$ and $D(0, 8)$ *(Souhag 2008)*
5	**Prove that :** The points $A(0, 1)$, $B(4, 5)$, $C(1, 8)$ and $D(-3, 4)$ are vertices of a rectangle and find its diagonal length. *(Souhag 2009)* « $5\sqrt{2}$ length units»
6	**Prove that :** The points $A(3, 3)$, $B(0, 3)$, $C(0, 0)$ and $D(3, 0)$ in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area. *(Luxor 2009)* « $3\sqrt{2}$ length units, 9 square units»
7	**Prove that :** The points $A(3, -1)$, $B(-4, 6)$ and $C(2, -2)$ lie on the same circle whose centre is $M(-1, 2)$, then find the circumference of the circle where $\pi = 3.14$ *(Alex. 2015 – Cairo 2015 – El-Sharkia 2013)* « 31.4 length units »
8	If $A(2, X)$ and $B(3, -1)$, $AB = \sqrt{17}$ length units, then find : X *(El-Dakahlia 2013)* « 3 or – 5 »
9	**Find the value of a in each of the following cases :** If the distance between the two points $(a, 7)$, $(-2, 3)$ equals 5 length unit. *(Luxor 2013)* « 1 or – 5 »

10 📖 If A (x , 3) , B (3 , 2) and C (5 , 1) and AB = BC , then find the value of x

(El-Beheira 2015 – Port Said 2014) « 5 or 1 »

If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3)

Find the coordinates of the point B *(Beni Suef 2014 – El-Beheira 2013)* « (7 , – 5) »

---

12 Find the value of each of a and b that satisfies that (2 a – 3 , a – b) is the midpoint of the line segment whose terminals (7 , –1) and (3 , 7) *(EL-Fayoum 2012)* « 4 , 1 »

$\overline{AB}$ is a diameter in a circle M , if B (8 , 11) and M (5 , 7) **Find :**

(1) The coordinates of A

(2) The perimeter of the circle. where (π = 3.14) *(Assiut 2014)* « A (2 , 3) , 31.4 length unit »

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14 ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) and C (0 , – 3)

Find the coordinates of the intersection point of its diagonals , then find the coordinates of the point D *(El-Monofia 2015 – Giza 2015)* « $\left(1\frac{1}{2} , -\frac{1}{2}\right)$, (– 1 , 4) »

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15 📖 **Prove that :** The points A (6 , 0) , B (2 , – 4) and C (– 4 , 2) are the vertices of a right-angled triangle at B , then find the coordinates of D that make the figure ABCD a rectangle. *(Kafr El-Sheikh 2014 – Assiut 2011)* « D (0 , 6) »



Lesson (3)

The slope of the straight line

Prelude

You studied before the slope of the straight line given two points on it.

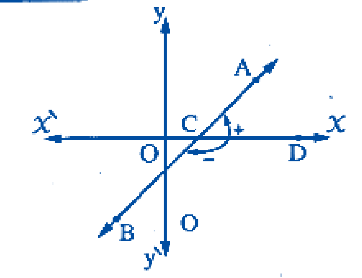
If A and B are two points in the coordinates plane where A (x_1, y_1) and B (x_2, y_2) , then :

The slope of the straight line $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$

The positive measure and the negative measure of an angle

In the opposite figure :

If $\overleftrightarrow{AB}$ intersects the x-axis at the point C , then $\overleftrightarrow{AB}$ makes two angles with the positive direction of the x-axis.



The slope of the straight line

Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the x-axis.

i.e. The slope of the straight line = tan θ where θ is the measure of the positive angle which the straight line makes with the positive direction of the x-axis.

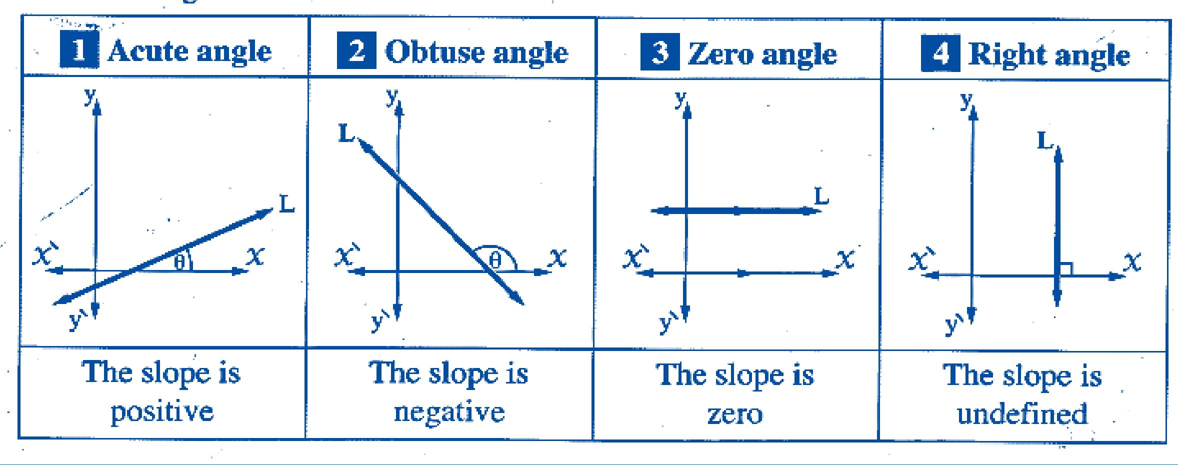
Notice that

The straight line passes through the two points (2 , 0) and (7 , 5) , then :

the slope of the straight line L $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-0}{7-2} = \dfrac{5}{5} = 1$

Remark

The angle which the straight line L makes with the positive direction of the x-axis takes one of the following cases :



1 Acute angle	2 Obtuse angle	3 Zero angle	4 Right angle
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

The relation between the two slopes of the two parallel straight lines

Also , we can deduce the opposite :

If $m_1 = m_2$, then $L_1 // L_2$

i.e. If the two straight lines have equal slopes, then the two straight lines are parallel.

The relation between the slopes of the two perpendicular (orthogonal) straight lines

If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively and $L_1 \perp L_2$, then $m_1 \times m_2 = -1$, unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines $= -1$

and vice versa :

Remark

If $L_1 \perp L_2$, the slope of L_1 is m_1 and the slope of L_2 is m_2 , then $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$

For example :

- If the slope of the straight line L is 2, then the slope of the perpendicular to it $= -\frac{1}{2}$
- If the slope of the straight line L is $-\frac{2}{3}$, then the slope of the perpendicular to it $= \frac{3}{2}$

Remarks to solve the problems on quadrilateral

- **To prove that a quadrilateral is a trapezium, we prove that :**
  Two opposite sides are parallel and the other two sides are not parallel.
- **To prove that a quadrilateral is a parallelogram, we prove only one of the following properties :**
  1. Each two opposite sides are parallel.
  2. Each two opposite sides are equal in length.
  3. Two opposite sides are parallel and equal in length.
  4. The two diagonals bisect each other.
- **To prove that a quadrilateral is a rectangle, rhombus or square, we prove at first that the quadrilateral is a parallelogram, then :**

– To prove that the parallelogram is a rectangle, we prove only one of the following two properties :

1. Two adjacent sides are perpendicular.
2. The two diagonals are equal in length.

– To prove that the parallelogram is a rhombus, we prove only one of the following two properties :

1. Two adjacent sides are equal in length.
2. The two diagonals are perpendicular.

– To prove that the parallelogram is a square, we prove only one of the following properties :

1. Two adjacent sides are perpendicular and equal in length.
2. Two adjacent sides are perpendicular and its diagonals are perpendicular.
3. Two diagonals are equal in length and perpendicular.
4. Two adjacent sides are equal in length and its two diagonals are equal in length.

Examples

1	Find the slope of the straight line which is perpendicular to the straight line which passes through the two points A (2 , – 3) , B (3 , 5) *(Matrouh 2009)* « $-\frac{1}{8}$ »
2	**Prove that :** The straight line which passes through the two points $(4 , 3\sqrt{3})$ and $(5 , 2\sqrt{3})$ is perpendicular to the straight line which makes an angle of measure 30° with the positive direction of X-axis. *(El-Beheira 2013)*
3	**Prove that :** The points A (1 , 1) , B (2 , 3) and C (0 , – 1) are collinear. *(Cairo 2013)*
4	The triangle whose vertices are A (3 , –1) , B (X , 3) and C (5 , 3) is a right-angled triangle at A , find the value of X *(Cairo 2011)* « –5 »
5	🕮 **Prove that :** The points A (– 1 , 1) , B (0 , 5) , C (4 , 2) and D (5 , 6) are the vertices of the parallelogram ABDC *(Luxor 2012)*

The solutions

Problem [1]

$\because m_1 = \frac{5+3}{3-2} = 8$ $\quad \because m_1 \times m_2 = -1$

$\therefore m_2 = -\frac{1}{8}$

Problem [2]

$\because m_1 = \frac{2\sqrt{3}-3\sqrt{3}}{5-4} = -\sqrt{3}$, $m_2 = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore m_1 \times m_2 = -1$

$\therefore$ The two straight lines are perpendicular

Problem [3]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{3-1}{2-1} = \frac{2}{1} = 2$

, the slope of $\overleftrightarrow{BC} = \frac{-1-3}{0-2} = \frac{-4}{-2} = 2$

$\because$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

$\because$ B is a common point between the two straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B , C are collinear points

Problem [4]

$\because \Delta ABC$ is a right-angled triangle at A

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{AC}$, the slope of $\overleftrightarrow{AC} = \frac{3+1}{5-3} = 2$

$\therefore$ The slope of $\overleftrightarrow{AB} = -\frac{1}{2}$

, $\because$ the slope of $\overleftrightarrow{AB} = \frac{3+1}{X-3} = \frac{4}{X-3}$

$\therefore \frac{4}{X-3} = -\frac{1}{2}$ $\quad \therefore X - 3 = -8$

$\therefore X = -5$

Problem [5]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{5-1}{0+1} = 4$

, the slope of $\overleftrightarrow{CD} = \frac{6-2}{5-4} = 4$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AC} = \frac{2-1}{4+1} = \frac{1}{5}$

, the slope of $\overleftrightarrow{BD} = \frac{6-5}{5-0} = \frac{1}{5}$

$\therefore$ The slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{BD}$

$\therefore \overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$ (2)

From (1) and (2) :

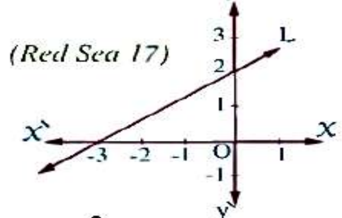
$\therefore$ The figure ABDC is a parallelogram

EX. (1): Choose the correct answer:

Exercises

1 **In the opposite figure :** *(Red Sea 17)*

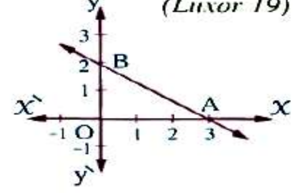
The slope of the straight line L equals ....



(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

2 **In the opposite figure :** *(Luxor 19)*

The slope of $\overleftrightarrow{AB}$ = ...



(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

3 The slope of the straight line that makes with the positive direction of the X-axis a positive angle of measure θ equals .... *(Giza 17)*

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\frac{\sin\theta}{\cos\theta}$ (d) $\sin\theta + \cos\theta$

4 If the slope of a straight line is more than zero , then the type of the positive angle which it makes with the positive direction of X-axis is .... *(Damietta 11)*

(a) zero. (b) acute. (c) right. (d) obtuse.

5 If m_1 and m_2 are the slopes of two perpendicular straight lines , then .... *(Qena 12)*

(a) $m_1 = m_2$ (b) $m_1 = -m_2$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$

6 If m_1 and m_2 are the slopes of two parallel straight lines , then ....

(a) $m_1 - m_2 = 0$ (b) $m_1 + m_2 = 0$ (c) $m_1 m_2 = 0$ (d) $m_1 - m_2 \neq 0$

7 The straight line that passes through the two points (0 , 0) and (2 , 3) is parallel to the straight line whose slope is ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

8 If the straight line L is perpendicular to the straight line which passes through the two points (– 1 , 2) and (0 , 5) , then the slope of the straight line L = ...

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

9 If m_1 and m_2 are the slopes of two perpendicular straight lines and $m_1 = 0.75$, then m_2 = ... *(El-Sharkia 13)*

(a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $\frac{3}{4}$

10 If the two straight lines whose slopes are $-\frac{2}{3}$ and $\frac{k}{2}$ are parallel , then k = ... *(Alex. 17 – Matrouh 19)*

(a) $\frac{-3}{4}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-4}{3}$

11 If the straight line which passes through the two points (X , 5) and (2 , 3) is parallel to the straight line which passes through the two points (3 , 4) and (5 , 2) , then X = ...

(a) 2 (b) – 2 (c) zero (d) 1

12 The straight line which passes through the two points (–1 , –1) and (4 , 4) makes with the positive direction of X-axis a positive angle of measure .... *(El-Monofia 15 – North Sinai 17)*

(a) 30° (b) 45° (c) 60° (d) 135°

13 The straight line which passes through two points (k , 0) and (0 , 4) is perpendicular to a straight line which makes an angle of measure 45° with the positive direction of X-axis , then k = ... *(Aswan 13)*

(a) 4 (b) –4 (c) 1 (d) –1



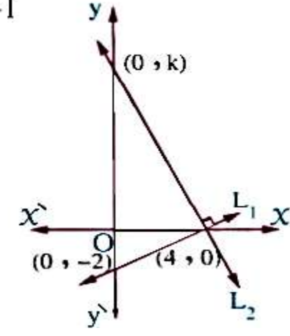
14 **In the opposite figure :**

If $L_1 \perp L_2$

, then k = ...

(a) 2 (b) 4

(c) 6 (d) 8



EX. (2): Answer the following:

1 **Prove that :** The straight line passing through the two points (2 , – 1) and (6 , 3) is parallel to the straight line that makes an angle of measure 45° with the positive direction of the X-axis. *(Kafr El-Sheikh 2011)*

2 If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle whose measure is 45° , then find k if the two straight lines L_1 and L_2 are :

(1) parallel (2) perpendicular *(Aswan 2014)* « 0 , 2 »

3 If the points (0 , 1) , (A , 3) and (2 , 5) are located on one straight line. Then find the value of A *(El-Gharbia 2014)* « 1 »

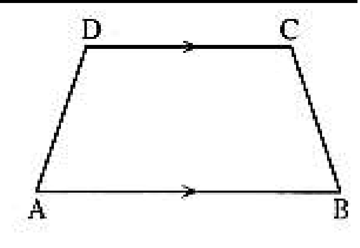
4 If A (– 1 , – 1) , B (2 , 3) and C (6 , 0) , **prove that :** the triangle ABC is a right-angled triangle at B *(Suez 2014)*

5 Prove by using the slope that the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are the vertices of the rectangle ABCD *(Beni Suef 2013)*

6 **In the drawn figure :**

ABCD is a trapezoid where $\overline{AB} // \overline{CD}$, A (9 , – 2) , B (3 , 2) , C (X , – X) and D (4 , – 3)

Find the coordinates of the point C



(Alex. 2014) « (1 , – 1) »

Explaining

Lesson (4)

The equation of the straight line given its slope and the intercepted part of y-axis

First — Finding the slope of a straight line and the length of the intercepted part from y-axis.

If the equation of a straight line in the form : $y = m x + c$, then :

- The slope of the straight line = m
- The length of the intercepted part from y-axis = $|c|$
  and it passes through the point (0 , c)

For Example : -

- The straight line whose equation is $y = \frac{1}{2} x + 7$
  its slope = $\frac{1}{2}$
  and the intercepted part from y-axis = 7 length units and passes through the point (0 , 7)
- The straight line whose equation is $y = 3 x - 5$, its slope = 3
  and cuts from the negative side of y-axis a part of 5 length units and passes through the point (0 , – 5)

Remarks

If the equation of a straight line in the form : $a x + b y + c = 0$

, then the slope of the straight line = $\frac{-\text{coefficient of } x}{\text{coefficient of } y}$

and the straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis = $\left|\frac{-c}{b}\right|$

For Example : -

1 The straight line whose equation : $x - 2 y + 3 = 0$

Its slope = $\frac{-1}{-2} = \frac{1}{2}$ and cut y-axis at the point $\left(0 , \frac{3}{2}\right)$

i.e. The straight line intercepts a part of length equals $\frac{3}{2}$ length unit from the positive side of y-axis.

2 The straight line whose equation : $3x + y + 4 = 0$

Its slope $= -3$ and cut y-axis at the point $(0, -4)$

i.e. The straight line intercepts a part of length equals 4 length units from the negative side of y-axis.

Second Finding the equation of the straight line given its slope and the length of intercepted part of y-axis

The straight line whose slope = m and cuts y-axis at the point $(0, c)$ its equation is in the form $\boxed{y = mx + c}$

1 The equation of the straight line which passes through the origin point O $(0, 0)$ is $\boxed{y = mx}$, where m is the slope of the straight line.

2 The equation of x-axis is $\boxed{y = 0}$

3 The equation of y-axis is $\boxed{x = 0}$

4 The equation of the straight line parallel to x-axis and passes through the point $(0, \ell)$ is $\boxed{y = \ell}$

5 The equation of the straight line which is parallel to y-axis and passes through the point $(k, 0)$ is $\boxed{x = k}$

EX. (1): Choose the correct answer:

Exercises

1. The slope of the straight line whose equation is : $3y = 2x - 5$ is ....
   (a) 3 (b) 2 (c) -5 (d) $\frac{2}{3}$

2. The straight line whose equation is : $3x - 3y + 5 = 0$ makes a positive angle with the positive direction of x-axis , its measure = ... *(El-Monofia 11)*
   (a) 30° (b) 45° (c) 60° (d) 90°

3. The straight line whose equation is : $2x - 3y - 6 = 0$ intercepts from y-axis a part of length .... units. *(El-Fayoum 13 – Cairo 14 – Qena 17 – El-Kalyoubia 18)*
   (a) -6 (b) -2 (c) $\frac{2}{3}$ (d) 2

4. The straight line whose equation is : $2x + 5y - 10 = 0$ cuts from x-axis a part of length .... units. *(El-Dakahlia 11)*
   (a) $\frac{2}{5}$ (b) 2 (c) $\frac{5}{2}$ (d) 5

5. The equation of the straight line which intercepts a part of length 4 units from the positive part of y-axis and parallel to the straight line : $y = 3x + 5$ is ....
   (a) $y = 3x + 4$ (b) $y = 4x + 3$ (c) $y = 3x - 4$ (d) $y = -3x + 4$

6. The two straight lines : $y = 3x - 5$ and $2y = 6x + 5$ are ....
   (a) parallel (b) coincident
   (c) intersecting and not perpendicular (d) perpendicular

7. If the two straight lines : $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are perpendicular , then k = ... *(El-Beheira 15 – Giza 16 – Red Sea 19)*
   (a) -4 (b) -3 (c) 3 (d) 4

8. If the two straight lines : $x + y = 5$ and $kx + 2y = 0$ are parallel , then k = ... *(El-Dakahlia 15 – Souhag 16 – Qena 17 – El-Menia 19)*
   (a) -2 (b) -1 (c) 1 (d) 2

9. If the straight line whose equation is : $y = kx + 5$ is parallel to x-axis , then k = ... *(El-Gharbia 18)*
   (a) 0 (b) 1 (c) 2 (d) 3

10. The two straight lines : $y = ax + b$ and $y = cx + d$ are perpendicular , then .... $= -1$ *(El-Gharbia 08 – Souhag 16)*
    (a) $a \times d$ (b) $b \times c$ (c) $a \times c$ (d) $b \times d$

11. The straight line passing through the two points (5 , 4) and (1 , 5) is perpendicular to the straight line ....
    (a) $4x = 3 - 4y$ (b) $5y + x = 4$ (c) $y = 4x$ (d) $x + 2y = 4$

12. The slope of the straight line whose equation is : $3y = ax - 5$ and passes through the point (20 , 5) is ....
    (a) -1 (b) 1 (c) -2 (d) $\frac{1}{3}$

13. If the straight line whose equation is : $ax + (2 - a)y = 5$ is parallel to the straight line which passes through (1 , 4) , (3 , 5) , then a = ...
    (a) 3 (b) -2 (c) 6 (d) 4

14 The area of the triangle in square units which is bounded by the straight lines $3x - 4y = 12$, $x = 0$, $y = 0$ equals .... *(El-Monofia 12 – El-Kalyoubia 15)*

(a) 6 (b) 7 (c) 12 (d) – 6

15 In the opposite figure : *(El-Monofia 17)*

If the area of Δ AOB = 9 square unit.

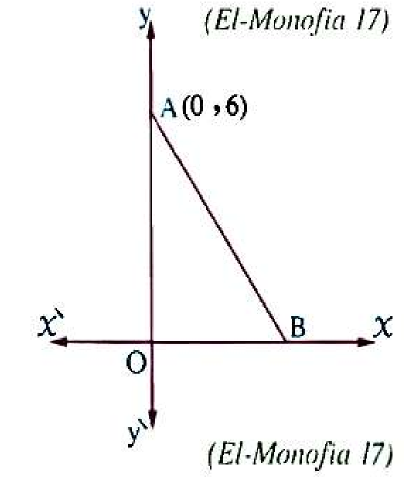
, then the equation of the straight line $\overleftrightarrow{AB}$ = ...

(a) $y = 2x + 6$

(b) $y = 6 - 2x$

(c) $y = 2x - 6$

(d) $y = \frac{1}{2}x - 6$



16 In the opposite figure : *(El-Monofia 17)*

If ABCD is a square

, the equation of the straight line

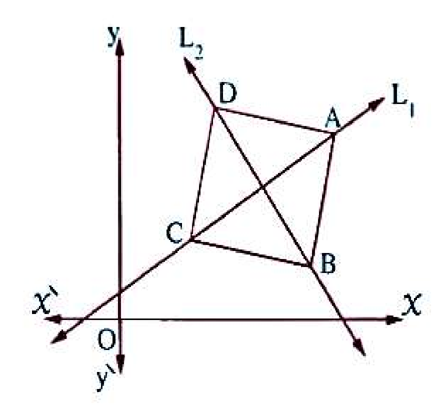
L_1 is : $y = \frac{2}{3}x + 1$

and the equation of the straight line

L_2 is : $y = kx + 14$, then k = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$



EX. (2): Answer the following:

1	**Find the equation of the straight line if :** 📖 Its slope = 2 and intercepts from the positive part of y-axis 7 units. *(Suez 2015)*
2	**Find the equation of the straight line :** Which cuts a part of length 3 units from the negative part of y-axis and is parallel to the line whose equation : $2x - 3y = 6$ *(El-Beheira 2011)*
3	Which passes through the point $(2, -1)$ and its slope equals 2 *(El-Kalyoubia 2011)*
4	Passing through the point $(-2, 3)$ and perpendicular to the straight line whose equation : $y = \frac{1}{2}x - 5$ *(El-Dakahlia 2013)*
5	📖 Passing through the point $(3, -5)$ and it is parallel to the straight line : $x + 2y - 7 = 0$ *(Alexandria 2015)*
6	Which passes through the point $(3, 2)$ and parallel to the straight line passing through the two points $(5, 6)$ and $(-1, 2)$ *(Helwan 2009)*
7	Passing through the point $(1, 2)$ and perpendicular to the straight line passing through the two points $A(2, -3)$ and $B(5, -4)$ *(Red Sea 2013 – El-Gharbia 2014)*
8	Passing through the point $(2, -2)$ and perpendicular to the straight line which makes an angle of measure $45°$ with the positive direction of x-axis *(Luxor 2011)*
9	📖 Which passes through the two points $(2, -1)$ and $(1, 1)$ *(El-Gharbia 2013)*
10	📖 Which passes through the two points $(4, 2)$ and $(-2, -1)$ then prove that it passes through the origin point. *(Suez 2015 – Dakahlia 2012)*
11	Which passes through the midpoint of the line segment $\overline{AB}$ where $A(3, 6)$ and $B(-1, 4)$ and perpendicular to the straight line whose equation is $2y - 4x + 11 = 0$ *(Cairo 2009)*
12	**Prove that :** The straight line $\overleftrightarrow{AB}$ is parallel to the straight line whose equation : $x - 2y + 8 = 0$ where $A(2, 3)$ and $B(-2, 1)$ *(El-Fayoum 2011)*

13

Prove that : The straight line whose equation : $2X + y + 8 = 0$ is perpendicular to the straight line passing through A (2 , 3) and B (– 2 , 1) *(Aswan 2012)*

14

If the straight line whose equation : $2X - 3y - 6 = 0$ cuts the X-axis at point A and the y-axis at point B , find : *(El-Sharkia 2013)*

(1) The coordinates of two points A and B

(2) The equation of the straight line passing through the midpoint of $\overline{AB}$ and parallel to the y-axis.

15

If the straight line whose equation : $aX + 2y - 3 = 0$ is parallel to the straight line which passes through the two point (2 , 3) , (1 , 5) which lie on the same plane , then find the value of a *(Souhag 2013)* « 4 »

16

Find the equation of the axis of symmetry of $\overline{XY}$, where X (3 , – 2) and Y (– 5 , 6) *(El-Dakahlia 2012 – Port Said 2014)*

17

Find the equation of the straight line which intercepts from the positive parts of the coordinate axes «x-axis and y-axis» two parts of lengths 4 and 9 length unit respectively. *(Assiut 2012)*

18

The opposite table represents a linear relation :

(1) Find the equation of the straight line.

(2) Find the length of the intercepted part from y-axis.

(3) Find the value of a

X	1	2	3
$y = f(X)$	1	3	a

(Alexandria 2015 – El-Kalyoubia 2013)

19

In the opposite figure :

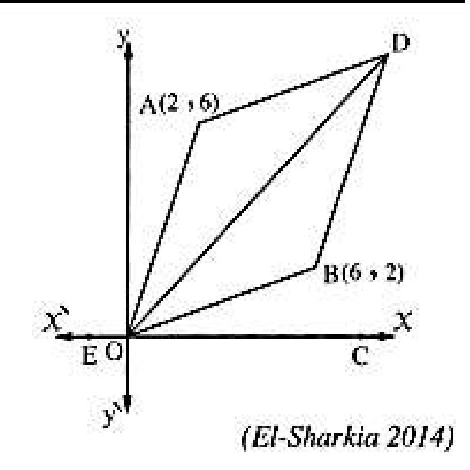
The points A (2 , 6) , O (0 , 0) , B (6 , 2) and D are vertices of the rhombus.

Find :

(1) The coordinates of the point D

(2) The equation of the straight line $\overleftrightarrow{OD}$

(3) m (∠ DOE)



(El-Sharkia 2014)


Guide Answers


Algebra

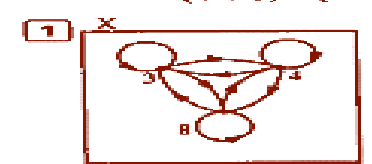
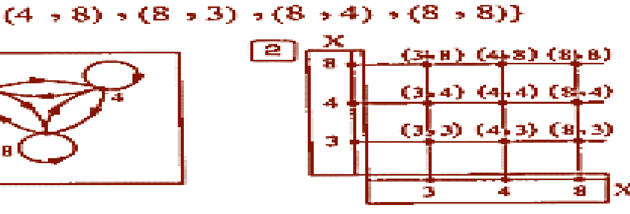


Lesson 1

Ex.(1): Choose

1	B	6	C	11	D	16	C	21	A	26	B	31	B	36	D
2	B	7	A	12	B	17	C	22	D	27	D	32	B	37	D
3	D	8	C	13	B	18	C	23	D	28	C	33	C	38	A
4	D	9	A	14	B	19	C	24	A	29	B	34	C	39	A
5	A	10	C	15	D	20	D	25	D	30	C	35	B	40	D

Ex.(2): Answer the following

1	a-7=-2 , a=5 , b^3 -1=26 , b^3=27 , b=$\sqrt[3]{27} = 3$
2	X-1 =8 , x = 9 , y + 3 = 11 , y = 8 . then:$\sqrt{X+2y} = \sqrt{9+2\times 8} = 5$
3	X^2 = 1 , X = $\pm\sqrt{1} = \pm 1$, $y^3 = 27$, $y = \sqrt[3]{27} = 3$. $then$: $\sqrt{y-x} = \sqrt{3-(-1)} = 4$
4	$2x = 8$, $x = 4$, $y + 1 = 4$, $y = 3$. $then$: $\sqrt{x^2+y^2} = \sqrt{4^2+3^2} = 5$
5	[1] $X \times Y = \{(2,4),(2,0),(-1,4),(-1,0)\}$ [2] $Y \times Z = \{(4,4),(4,5),(4,-2),(0,4),(0,5),(0,-2)\}$ [3] $X^2 = \{(2,2),(2,-1),(-1,2),(-1,-1)\}$ [4] $n(X \times Z) = 2 \times 3 = 6$ [5] $n(Y^2) = 2 \times 2 = 4$ [6] $n(Z^2) = 3 \times 3 = 9$
6	[1] $X = \{1\}$, $Y = \{1,3,5\}$ [2] $Y \times X = \{(1,1),(3,1),(5,1)\}$ [3] $Y^2 = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$
7	$X^2 = \{(3,3),(3,4),(3,8),(4,3),(4,4),(4,8),(8,3),(8,4),(8,8)\}$ [1]  The arrow diagram [2]  The Cartesian diagram
8	[1] $X \times (Y \cap Z) = \{3,4\} \times \{5\} = \{(3,5),(4,5)\}$ [2] $(X - Y) \times Z = \{3\} \times \{6,5\} = \{(3,6),(3,5)\}$ [3] $(X - Y) \times (Y - Z) = \{3\} \times \{4\} = \{(3,4)\}$
9	$X \times Y = \{(1,3),(1,4),(1,5),(2,3),(2,4),(2,5)\}$ [1] X: 1, 2 → Y: 3, 4, 5  The arrow diagram [2] Y: 5, 4, 3 — (1,5) (2,5), (1,4) (2,4), (1,3) (2,3) — X: 1, 2 The Cartesian diagram

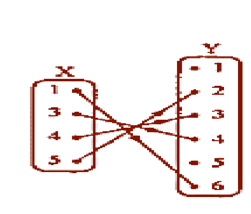
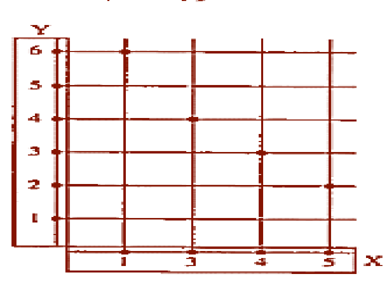
10	(1)X={2} , y = {3,2,4} (2) X × (X ∩ y) = {2} × {2} = {(2,2)}
11	$A \in X \times X$ $B \in X \times X$ $C \notin X \times X$ $D \in X \times X$
12	(1)Y×Z = {4,1} × {15} = {(4,15) , (1,15)} (2) $n(X^2) = 2^2 = 4$ (3) (X∩Z) × y = {15} × {4,1}={(15,4) , (15,1)}
13	A Lies on the first quadrant B Lies on the fourth quadrant C Lies on the second quadrant D Lies on the second quadrant E Lies on the third quadrant M Lies on y-axis K Lies on X-axis
14	**First :** 1 X × Y = {(1 , 2) , (1 , 3)} 2 Y × Z = {(2 , 2) , (2 , 5) , (2 , 6) , (3 , 2) , (3 , 5) , (3 , 6)} 3 X × Z = {(1 , 2) , (1 , 5) , (1 , 6)} 4 Y^2 = {(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)} **Second :** (X × Y) ∪ (Y × Z) = {(1 , 2) , (1 , 3) , (2 , 2) , (2 , 5) , (2 , 6) , (3 , 2) , (3 , 5) , (3 , 6)} **Third :** X × (Y ∩ Z) = {1} × {2} = {(1 , 2)} **Fourth :** (X × Y) ∩ (X × Z) = {(1 , 2)} **Fifth :** (Z − Y) × (X ∪ Y) = {5 , 6} × {1 , 2 , 3} = {(5 , 1) , (5 , 2) , (5 , 3) , (6 , 1) , (6 , 2) , (6 , 3)}

Lesson 2

Ex.(1): Choose

1	A	6	A	11	D	16	A	21	B	26	A
2	C	7	C	12	D	17	B	22	C	27	D
3	B	8	A	13	D	18	D	23	D		
4	B	9	D	14	C	19	C	24	A		
5	C	10	B	15	D	20	B	25	D		

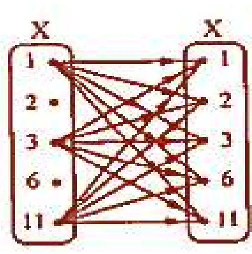
Ex.(2): Answer the following

1	$R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$  
2	 $R = \{(0, 1), (0, 3), (0, 5), (0, 6), (1, 1), (1, 3), (1, 5), (1, 6), (4, 1), (4, 3)\}$ R is not a function because $0 \in X$, $1 \in X$, $4 \in X$ each of them has more than one image in Y also $7 \in X$ has no image in Y
3	$R = \{(2, 4), (2, 5), (2, 6), (2, 7), (2, 9), (4, 4), (4, 5), (4, 6), (4, 7), (4, 9), (5, 5), (5, 6), (5, 7), (5, 9), (7, 7), (7, 9)\}$ Represent by yourself.
4	$R = \{(1, 1), (2, 8)\}$ Represent by yourself.
5	$R = \{(2, 10), (2, 16), (2, 24), (2, 30), (5, 10), (5, 30), (8, 16), (8, 24)\}$ R is not a function because $2 \in X$ has more than one image in Y also $5 \in X$, $8 \in X$ each of them has two images in Y Represent by yourself.
6	$R = \{(2, 6), (2, 8), (2, 10), (3, 6), (3, 15), (4, 8)\}$

7

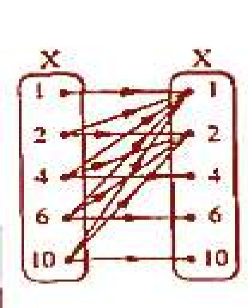
$R = \{(1 , 1) , (1 , 2) , (1 , 3) , (1 , 6)$
$, (1 , 11) , (3 , 1) , (3 , 2) , (3 , 3)$
$, (3 , 6) , (3 , 11) , (11 , 1) , (11 , 2)$
$, (11 , 3) , (11 , 6) , (11 , 11)\}$

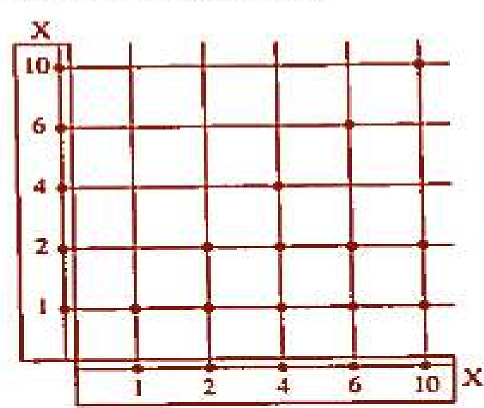
R is not a function because each of $1 \in X , 3 \in X , 11 \in X$ has more than one image in X also each of $2 \in X , 6 \in X$ has no image in X



8

$R = \{(1 , 1) , (2 , 1) , (2 , 2) , (4 , 1) ,$
$(4 , 2) , (4 , 4) , (6 , 1) , (6 , 2) ,$
$(6 , 6) , (10 , 1) , (10 , 2) , (10 , 10)\}$





R is not a function because each of $2 \in X , 4 \in X ,$ $6 \in X$ and $10 \in X$ has more than one image in X

Lesson 3

Ex.(1): Choose

1	C	6	C	11	C
2	C	7	C		
3	D	8	D		
4	D	9	C		
5	D	10	D		

Ex.(2): Answer the following

1

$\because f(2) = 2 \times 2 - 1 = 3$, $f(1) = 2 \times 1 - 1 = 1$

$\therefore f(2) - 3f(1) = 3 - 3 \times 1 = \text{zero}$

2

$\because f(2) = 2 \times (2)^2 - 5 \times 2 + 2 = \text{zero}$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 5 \times \frac{1}{2} + 2 = \text{zero}$

$\therefore f(2) = f\left(\frac{1}{2}\right)$

3

$\because f(a) = b$ $\quad \therefore b = a^2 + b$ $\quad \therefore a^2 = 0$

$\therefore a = 0$ $\quad \therefore a b^2 + 5 = 0 \times b^2 + 5 = 5$

4

$\because f(1+\sqrt{6}) = (1+\sqrt{6})^2 - 2(1+\sqrt{6}) - 5$

$= 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} - 5 = \text{zero}$

$f(1-\sqrt{6}) = (1-\sqrt{6})^2 - 2(1-\sqrt{6}) - 5$

$= 1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} - 5 = \text{zero}$

$\therefore f(1+\sqrt{6}) = f(1-\sqrt{6}) = \text{zero}$

5

[1] $f(\sqrt{2}) + 3g(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

[2] $\because f(3) = (3)^2 - 3 \times 3 = 9 - 9 = \text{zero}$

$g(3) = 3 - 3 = \text{zero}$

$\therefore f(3) = g(3) = \text{zero}$

6

	Degree	$f(-2)$	$f(0)$	$f\left(\frac{1}{2}\right)$
[1]	First	7	3	2
[2]	Second	zero	-4	$-3\frac{3}{4}$

7

[1] The domain $= \{1, 2, 3, 4, 5\}$

[2] The range $= \{3, 5, 7, 9, 11\}$

[3] The rule of the function f is : $f(x) = 2x + 1$

8

The polynomial functions are numbers ①, ②, ④

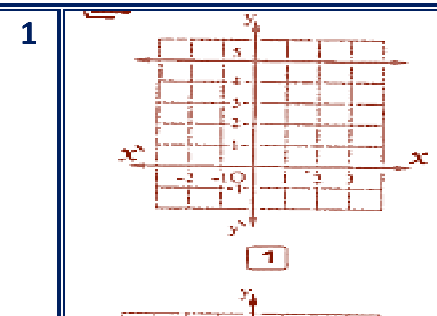
Lesson 4

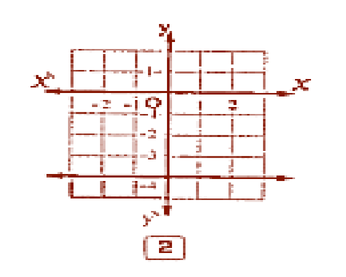
Ex.(1): Choose

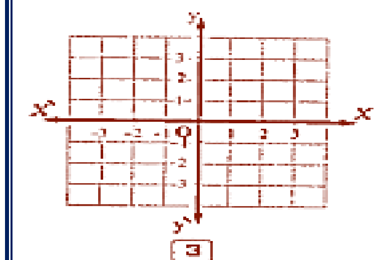
1	B	6	D	11	A	16	C
2	A	7	A	12	B	17	D
3	C	8	C	13	D	18	A
4	C	9	C	14	A	19	C
5	C	10	D	15	C	20	A

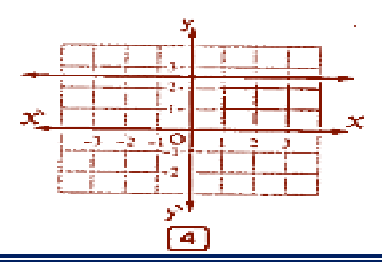
Ex.(2): Answer the following

1





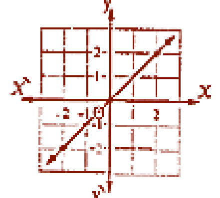




2


1 $f(x) = x$

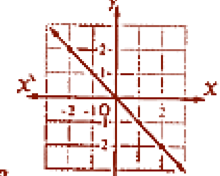
x	-2	zero	2
$f(x)$	-2	zero	2



The straight line representing the function intersects The two coordinate axes at the origin point O (0 , 0)

2 $f(x) = -x$

x	-2	zero	2
$f(x)$	2	zero	-2



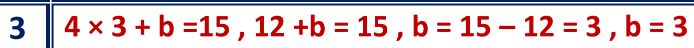
The straight line representing the function intersects The two coordinate axes at the origin point O (0 , 0)

3 $f(x) = 3x$

x	-1	zero	1
$f(x)$	-3	zero	3

The straight line representing the function intersects The two coordinate axes at the origin point O (0 , 0)

3


$4 \times 3 + b = 15$, $12 + b = 15$, $b = 15 - 12 = 3$, $b = 3$

4

$F(-2) = (-2)^2 - (-2) + 3 = 9$, $f(\text{zero}) = (0)^2 - (0) + 3 = 3$, $f(\sqrt{3}) = (\sqrt{3})^2 - \sqrt{3} + 3 = 6 - \sqrt{3}$



5	[1] Let A (x , 0) , ∵ A (x , 0) belongs to the straight line of the function f $\therefore 4 - 2x = 0$ $\therefore -2x = -4$ $\therefore x = \frac{-4}{-2} = 2$ ∴ A (2 , 0) , let B (0 , y) , ∵ B (0 , y) belongs to the straight line of the function f $\therefore 4 - 2 \times 0 = y$ $\therefore y = 4$ ∴ B (0 , 4) [2] Area of Δ AOB $= \frac{1}{2} \times 2 \times 4 = 4$ square unit
6	∵ A O = 4 units ∴ A (0 , 4) ∵ A (0 , 4) belongs to the curve of the function f ∴ A satisfies the equation of the curve $\therefore 4 = m - (0)^2$ $\therefore m = 4$ (the first req.) ∵ The curve of the function intersects x-axis at the two points B and C $\therefore 0 = 4 - x^2$ $\therefore x^2 = 4$ $\therefore x = 2$ or -2 ∴ B = (2 , 0) , C = (−2 , 0) (the second req.) ∴ BC = 4 units The area of Δ ABC $= \frac{1}{2} \times 4 \times 4 = 8$ square units (the third req.)

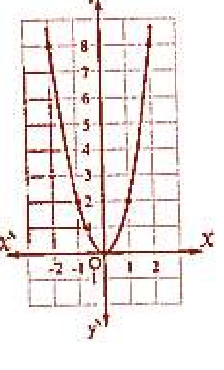
7

[1] $f(x) = 2x^2$

x	−2	−1	0	1	2
$f(x)$	8	2	0	2	8

From the graph :

- The vertex of the curve is (0 , 0)
- The equation of the line of symmetry is $x = 0$
- The minimum value = zero

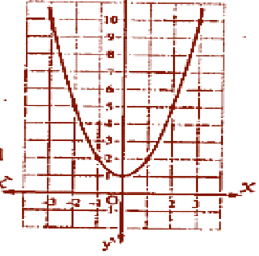


[2] $f(x) = x^2 + 1$

x	−3	−2	−1	0	1	2	3
$f(x)$	10	5	2	1	2	5	10

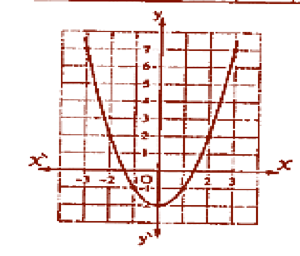
From the graph :

- The vertex of the curve is (0 , 1)
- The equation of the line of symmetry is $x = 0$
- The minimum value = 1



[3] $f(x) = x^2 - 2$

x	−3	−2	−1	0	1	2	3
$f(x)$	7	2	−1	−2	−1	2	7



From the graph we find that :

- The vertex of the curve is (0 , −2)
- The equation of the line of symmetry is $x = 0$
- The minimum value = −2

[4] $f(x) = 2 - x^2$

x	−3	−2	−1	0	1	2	3
$f(x)$	−7	−2	1	2	1	−2	−7

Represent by yourself.

From the graph we find that :

- The vertex of the curve is (0 , 2)
- The equation of the line of symmetry is $x = 0$
- The maximum of value = 2

[5] $f(x) = x^2 - 2x$

x	−2	−1	0	1	2	3	4
$f(x)$	8	3	0	−1	0	3	8

Represent by yourself.

From the graph we find that :

- The vertex of the curve is (1 , −1)
- The equation of the line of symmetry is : $x = 1$
- The minimum value = −1

[6] $f(x) = x^2 + 2x + 1$

x	−4	−3	−2	−1	0	1	2
$f(x)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph we find that :

- The vertex of the curve is (−1 , 0)
- The equation of the line of symmetry is $x = -1$
- The minimum value = 0

[7] $f(x) = (x - 2)^2 = x^2 - 4x + 4$

x	−1	0	1	2	3	4	5
$f(x)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph we find that :

- The vertex of the curve is (2 , 0)
- The equation of the line of symmetry is $x = 2$
- The minimum value = zero



Lessons 1,2

Ex.(1): Choose

1	B	6	A	11	D	16	C
2	C	7	A	12	D	17	D
3	D	8	A	13	D	18	A
4	B	9	B	14	B	19	A
5	D	10	C	15	B		

Ex.(2): Answer the following

1	$\because \frac{x}{y} = \frac{2}{3}$ $\quad \therefore x = 2m , y = 3m$ $\therefore \frac{3x + 2y}{6y - x} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$
2	$\because \frac{a+b}{b} = \frac{c+d}{d}$ $\quad \therefore d(a+b) = b(c+d)$ $\therefore ad + bd = bc + bd$ $\quad \therefore ad = bc$ $\therefore \frac{a}{b} = \frac{c}{d}$ $\therefore a , b , c , d$ are proportional. **Another solution :** $\frac{a+b}{b} = \frac{c+d}{d}$ $\quad \therefore \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$ $\therefore \frac{a}{b} + 1 = \frac{c}{d} + 1$ $\quad \therefore \frac{a}{b} = \frac{c}{d}$ $\therefore a , b , c , d$ are proportional.
3	$\because \frac{a}{b} = \frac{3}{5}$ $\quad \therefore a = 3m , b = 5m$ $\therefore \frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = 3$
4	Let the number be x $\quad \therefore \frac{7+x}{11+x} = \frac{2}{3}$ $\therefore 21 + 3x = 22 + 2x$ $\quad \therefore x = 1$ $\therefore$ The required number is 1
5	Let the number be x $\quad \therefore \frac{49 - 3x}{69 - 3x} = \frac{2}{3}$ $\therefore 147 - 9x = 138 - 6x$ $\quad \therefore 3x = 9$ $\therefore x = 3$ $\therefore$ The required number = 3
6	Let the number be x $\quad \therefore \frac{7 + x^2}{11 + x^2} = \frac{4}{5}$ $\therefore 35 + 5x^2 = 44 + 4x^2$ $\quad \therefore x^2 = 9$ $\therefore x = \pm 3$ $\therefore$ The required number is 3 or -3
7	Let the number be x $\quad \therefore \frac{5 + x^2}{11 + x^2} = \frac{3}{5}$ $\therefore 25 + 5x^2 = 33 + 3x^2$ $\quad \therefore 2x^2 = 8$ $\therefore x^2 = 4$ $\therefore x = 2$ or $x = -2$ (refused) $\therefore$ The required number = 2

8

Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{3}{7}$ $\quad \therefore a = 3m , b = 7m$

$\therefore \frac{3m-5}{7m-5} = \frac{1}{3}$ $\quad \therefore 9m - 15 = 7m - 5$

$\therefore 2m = 10$ $\quad \therefore m = 5$

$\therefore$ The two numbers are 15 and 35

9

Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{2}{3}$ $\quad \therefore a = 2m , b = 3m$

$\therefore \frac{2m+7}{3m-12} = \frac{5}{3}$ $\quad \therefore 6m + 21 = 15m - 60$

$\therefore 81 = 9m$ $\quad \therefore m = 9$

$\therefore$ The two numbers are 18 and 27

10

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by -5 and the 3rd by 3 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-5b+3c}{4-15+12}$ = one of the given ratios.

$\therefore 2a - 5b + 3c$ = one of the given ratios.

11

$\because$ Let $\frac{a}{b} = \frac{c}{d} = m$ where $m > 0$

$\therefore a = bm , c = dm$

[1] L.H.S. $= \frac{3a+c}{5a-2c} = \frac{3bm+dm}{5bm-2dm} = \frac{m(3b+d)}{m(5b-2d)}$

$= \frac{3b+d}{5b-2d}$

= R.H.S.

From (1) and (2) : $\quad \therefore$ The two sides are equal.

[2] $\because \frac{3a-2c}{5a+3c} = \frac{3b-2d}{5b+3d}$ $\quad \therefore \frac{3a-2c}{3b-2d} = \frac{5a+3c}{5b+3d}$

$\because a = bm , c = dm$

$\therefore$ L.H.S. $= \frac{3bm-2dm}{3b-2d} = \frac{m(3b-2d)}{3b-2d} = m$ $\quad (1)$

, R.H.S. $= \frac{5bm+3dm}{5b+3d} = \frac{m(5b+3d)}{5b+3d} = m$ $\quad (2)$

From (1) and (2) : $\quad \therefore$ The two sides are equal.

12

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$

, multiplying the two terms of 1st ratio by 2 and the 2nd by -1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-b+5c}{4-3+20}$ = one of the given ratios.

$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3x}$

$\therefore 3x = 21$ $\quad \therefore x = 7$

13

$\because \frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$

, adding the antecedents and consequents of the three ratios $\therefore \frac{y+x+x+y}{x-z+y+z} = \frac{2(x+y)}{(x+y)} = 2$

= one of the given ratios.

$\therefore$ Each ratio = 2 unless $x + y = 0$

$\therefore \frac{x}{y} = 2$ $\quad \therefore x = 2y$

, $\frac{x+y}{z} = 2$ $\quad \therefore x + y = 2z$ $\quad \therefore 2y + y = 2z$

$\therefore 3y = 2z$ $\quad \therefore z = \frac{3}{2} y$

$\therefore x : y : z = 2y : y : \frac{3}{2} y = 4 : 2 : 3$

14

$\because \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$ where $m > 0$

$\therefore x = 3m, y = 4m, z = 5m$

[1] The left side $= \frac{2y - z}{3x - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m}$

$= \frac{3m}{6m} = \frac{1}{2}$

[2] $\because \sqrt{3x^2 + 3y^2 + z^2} = \sqrt{27m^2 + 48m^2 + 25m^2}$

$= \sqrt{100m^2} = 10m \qquad (1)$

$2x + y = 6m + 4m = 10m \qquad (2)$

From (1) and (2) we deduce that :

$\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

15

$\because \frac{x}{2a + b} = \frac{y}{2b - c} = \frac{z}{2c - a}$

, multiplying the two terms of the 1st ratio by 2 and adding the antecedents and consequents of the 1st and the 2nd ratios.

$\therefore \frac{2x + y}{4a + 2b + 2b - c} = \frac{2x + y}{4a + 4b - c}$

= one of the given ratios. (1)

, multiplying the terms of the 1st ratio by 2 and the 2nd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2x + 2y + z}{4a + 2b + 4b - 2c + 2c - a} = \frac{2x + 2y + z}{3a + 6b}$

= one of the given ratios. (2)

From (1) and (2) :

$\therefore \frac{2x + y}{4a + 4b - c} = \frac{2x + 2y + z}{3a + 6b}$

16

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ where $m > 0$

$\therefore a = bm, c = dm, c = fm$

[1] L.H.S. $= \frac{a + 5c}{b + 5d} = \frac{bm + 5dm}{b + 5d} = \frac{m(b + 5d)}{(b + 5d)} = m \ (1)$

R.H.S. $= \frac{c - 3e}{d - 3f} = \frac{dm - 3fm}{d - 3f} = \frac{m(d - 3f)}{d - 3f} = m \ (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

[2] L.H.S. $= \frac{2a + 7c - 4e}{2b + 7d - 4f} = \frac{2bm + 7dm - 4fm}{2b + 7d - 4f}$

$= \frac{m(2b + 7d - 4f)}{(2b + 7d - 4f)} = m \qquad (1)$

R.H.S. $= \frac{a - 8e}{b - 8f} = \frac{bm - 8fm}{b - 8f} = \frac{m(b - 8f)}{b - 8f} = m \qquad (2)$

From (1) and (2) : $\therefore$ The two sides are equal.

Lesson 3

Ex.(1): Choose

1 c	2 c	3 d	4 c	5 b
6 c	7 c	8 c	9 c	10 c

Ex.(2): Answer the following

1

[1] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a}{c}=\frac{cm^2}{c}=m^2$ (1) , $\frac{b^2}{c^2}=\frac{c^2m^2}{c^2}=m^2$ (2)

From (1) and (2) : $\therefore \frac{a}{c}=\frac{b^2}{c^2}$

Another solution :

$\because b^2=ac$ $\therefore \frac{b^2}{c^2}=\frac{ac}{c^2}=\frac{a}{c}=$ L.H.S.

[2] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{2a+3b}{2b+3c}=\frac{2cm^2+3cm}{2cm+3c}=\frac{cm(2m+3)}{c(2m+3)}=m$ (1)

, $\frac{a}{b}=\frac{cm^2}{cm}=m$ (2)

From (1) and (2) : $\therefore \frac{2a+3b}{2b+3c}=\frac{a}{b}$

[3] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a-b}{b-c}=\frac{cm^2-cm}{cm-c}=\frac{cm(m-1)}{c(m-1)}=m$ (1)

, $\frac{a+3b}{3c+b}=\frac{cm^2+3cm}{3c+cm}=\frac{cm(m+3)}{c(3+m)}=m$ (2)

From (1) and (2) : $\therefore \frac{a-b}{b-c}=\frac{a+3b}{3c+b}$

[4] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{c^2m^4+c^2m^2}{c^2m^2+c^2}=\frac{c^2m^2(m^2+1)}{c^2(m^2+1)}=m^2$ (1)

, $\frac{a}{c}=\frac{cm^2}{c}=m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{a}{c}$

Another solution :

$\because b^2=ac$

$\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{a^2+ac}{ac+c^2}=\frac{a(a+c)}{c(a+c)}=\frac{a}{c}=$ R.H.S.

[5] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \left(\frac{b-c}{a-b}\right)^2=\left(\frac{cm-c}{cm^2-cm}\right)^2=\left(\frac{c(m-1)}{cm(m-1)}\right)^2$

$=\frac{1}{m^2}$ (1)

, $\frac{c}{a}=\frac{c}{cm^2}=\frac{1}{m^2}$ (2)

From (1) and (2) : $\therefore \left(\frac{b-c}{a-b}\right)^2=\frac{c}{a}$

[6] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a^3+b^3}{b^3+c^3}=\frac{c^3m^6+c^3m^3}{c^3m^3+c^3}=\frac{c^3m^3(m^3+1)}{c^3(m^3+1)}=m^3$ (1)

, $\frac{a^2}{cb}=\frac{c^2m^4}{c\times cm}=m^3$ (2)

From (1) and (2) : $\therefore \frac{a^3+b^3}{b^3+c^3}=\frac{a^2}{cb}$

[7] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a^3-4b^3}{b^3-4c^3}=\frac{c^3m^6-4c^3m^3}{c^3m^3-4c^3}$

$=\frac{c^3m^3(m^3-4)}{c^3(m^3-4)}=m^3$ (1)

$\frac{b^3}{c^3}=\frac{c^3m^3}{c^3}=m^3$ (2)

From (1) and (2) : $\therefore \frac{a^3-4b^3}{b^3-4c^3}=\frac{b^3}{c^3}$

[8] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{2c^2-3c^2m^2}{2c^2m^2-3c^2m^4}$

$=\frac{c^2(2-3m^2)}{c^2m^2(2-3m^2)}=\frac{1}{m^2}$ (1)

, $\frac{c}{a}=\frac{c}{cm^2}=\frac{1}{m^2}$ (2) , $\frac{c^2}{b^2}=\frac{c^2}{c^2m^2}=\frac{1}{m^2}$ (3)

From (1) , (2) and (3) : $\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{c}{a}=\frac{c^2}{b^2}$

Another solution : $\because b^2=ac$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{2c^2-3ac}{2ac-3a^2}=\frac{c(2c-3a)}{a(2c-3a)}=\frac{c}{a}$

, $\frac{c^2}{b^2}=\frac{c^2}{ac}=\frac{c}{a}$

$\therefore \frac{2c^2-3b^2}{2b^2-3a^2}=\frac{c}{a}=\frac{c^2}{b^2}$

[9] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{c^2m^4+c^2m^3+c^2m^2}{c^2m^2+c^2m+c^2}$

$=\frac{c^2m^2(m^2+m+1)}{c^2(m^2+m+1)}=m^2$ (1)

, $\frac{a^2-b^2}{b^2-c^2}=\frac{c^2m^4-c^2m^2}{c^2m^2-c^2}=\frac{c^2m^2(m^2-1)}{c^2(m^2-1)}=m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a^2-b^2}{b^2-c^2}$

[10] Let $\frac{a}{b}=\frac{b}{c}=m$ $\therefore b=cm$, $a=cm^2$

$\therefore \frac{2a}{c}=\frac{2cm^2}{c}=2m^2$ (1)

, $\frac{a^2}{b^2}+\frac{b^2}{c^2}=m^2+m^2=2m^2$ (2)

From (1) and (2) : $\therefore \frac{2a}{c} = \frac{a^2}{b^2} + \frac{b^2}{c^2}$

Another solution : $\because b^2 = ac$

$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2a}{c} = \text{R.H.S.}$

[11] Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$$\therefore \frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = \frac{cm^2+cm+c}{c^{-1}m^{-2}+c^{-1}m^{-1}+c^{-1}}$$

$$= \frac{c(m^2+m+1)}{c^{-1}m^{-2}(1+m+m^2)}$$

$$= c \times cm^2 = c^2m^2 = b^2$$

[12] Let : $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$$\therefore \frac{ac}{b(b+c)} = \frac{cm^2 \times c}{cm(cm+c)} = \frac{c^2m^2}{c^2m(m+1)} = \frac{m}{m+1} \quad (1)$$

$$, \frac{a}{a+b} = \frac{cm^2}{cm^2+cm} = \frac{cm^2}{cm(m+1)} = \frac{m}{m+1} \quad (2)$$

From (1) and (2) : $\therefore \frac{ac}{b(b+c)} = \frac{a}{a+b}$

Another solution : $\because b^2 = ac$

$$\therefore \frac{ac}{b(b+c)} = \frac{ac}{b^2+bc} = \frac{ac}{ac+bc} = \frac{ac}{c(a+b)}$$

$$= \frac{a}{a+b} = \text{R.H.S}$$

2

[1] Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$$\therefore \frac{a-2b}{b-2c} = \frac{dm^3-2dm^2}{dm^2-2dm} = \frac{dm^2(m-2)}{dm(m-2)} = m \quad (1)$$

$$, \frac{3b+4c}{3c+4d} = \frac{3dm^2+4dm}{3dm+4d} = \frac{dm(3m+4)}{d(3m+4)} = m \quad (2)$$

From (1) and (2) : $\therefore \frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$

[2] Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$$\therefore \frac{3a+5c}{3b+5d} = \frac{3dm^3+5dm}{3dm^2+5d} = \frac{dm(3m^2+5)}{d(3m^2+5)} = m \quad (1)$$

$$, \frac{a-4c}{b-4d} = \frac{dm^3-4dm}{dm^2-4d} = \frac{dm(m^2-4)}{d(m^2-4)} = m \quad (2)$$

From (1) and (2) : $\therefore \frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$

[3] Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{3a-5c}{a-b+c} = \frac{3dm^3-5dm}{dm^3-dm^2+dm}$

$= \frac{dm(3m^2-5)}{dm(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$ (1)

$, \frac{3b-5d}{b-c+d} = \frac{3dm^2-5d}{dm^2-dm+d}$

$= \frac{d(3m^2-5)}{d(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$ (2)

From (1) and (2) : $\therefore \frac{3a-5c}{a-b+c} = \frac{3b-5d}{b-c+d}$

4 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a-d}{a+b+c} = \frac{dm^3-d}{dm^3+dm^2+dm}$

$= \frac{d(m^3-1)}{dm(m^2+m+1)} = \frac{(m-1)(m^2+m+1)}{m(m^2+m+1)}$

$= \frac{m-1}{m}$ (1)

$, \frac{a-2b+c}{a-b} = \frac{dm^3-2dm^2+dm}{dm^3-dm^2}$

$= \frac{dm(m^2-2m+1)}{dm^2(m-1)} = \frac{(m-1)^2}{m(m-1)} = \frac{m-1}{m}$ (2)

From (1) and (2) : $\therefore \frac{a-d}{a+b+c} = \frac{a-2b+c}{a-b}$

5 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{c^2-d^2}{a-c} = \frac{d^2m^2-d^2}{dm^3-dm} = \frac{d^2(m^2-1)}{dm(m^2-1)} = \frac{d}{m}$ (1)

$, \frac{bd}{a} = \frac{d^2m^2}{dm^3} = \frac{d}{m}$ (2)

From (1) and (2) : $\therefore \frac{c^2-d^2}{a-c} = \frac{bd}{a}$

6 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{d^2m^6-3d^2m^2}{d^2m^4-3d^2} = \frac{d^2m^2(m^4-3)}{d^2(m^4-3)}$

$= m^2$ (1)

$, \frac{b}{d} = \frac{dm^2}{d} = m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$

7 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{ab-cd}{b^2-c^2} = \frac{dm^3 \times dm^2 - dm \times d}{d^2m^4-d^2m^2} = \frac{d^2m^5-d^2m}{d^2m^2(m^2-1)}$

$= \frac{d^2m(m^4-1)}{d^2m^2(m^2-1)} = \frac{(m^2-1)(m^2+1)}{m(m^2-1)} = \frac{m^2+1}{m}$ (1)

$, \frac{a+c}{b} = \frac{dm^3+dm}{dm^2} = \frac{dm(m^2+1)}{dm^2} = \frac{m^2+1}{m}$ (2)

From (1) and (2) : $\therefore \frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

8 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a}{b+d} = \frac{dm^3}{dm^2+d} = \frac{dm^3}{d(m^2+1)} = \frac{m^3}{m^2+1}$ (1)

$, \frac{c^3}{c^2d+d^3} = \frac{d^3m^3}{d^2m^2 \times d + d^3} = \frac{d^3m^3}{d^3(m^2+1)} = \frac{m^3}{m^2+1}$ (2)

From (1) and (2) : $\therefore \frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$

9 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{d^2m^6+d^2m^4+d^2m^2}{d^2m^4+d^2m^2+d^2}$

$= \frac{d^2m^2(m^4+m^2+1)}{d^2(m^4+m^2+1)} = m^2$ (1)

$, \frac{ac}{bd} = \frac{dm^3 \times dm}{dm^2 \times d} = m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$

10 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{2a+3d}{3a-4d} = \frac{2dm^3+3d}{3dm^3-4d}$

$= \frac{d(2m^3+3)}{d(3m^3-4)} = \frac{2m^3+3}{3m^3-4}$ (1)

$, \frac{2a^3+3b^3}{3a^3-4b^3} = \frac{2d^3m^9+3d^3m^6}{3d^3m^9-4d^3m^6}$

$= \frac{d^3m^6(2m^3+3)}{d^3m^6(3m^3-4)} = \frac{2m^3+3}{3m^3-4}$ (2)

From (1) and (2) : $\therefore \frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$

11 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ where $m > 0$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a+5b}{b+5c} = \frac{dm^3+5dm^2}{dm^2+5dm} = \frac{dm^2(m+5)}{dm(m+5)} = m$ (1)

$, \sqrt{\frac{b}{d}} = \sqrt{\frac{dm^2}{d}} = \sqrt{m^2} = m$ (2)

From (1) and (2) : $\therefore \frac{a+5b}{b+5c} = \sqrt{\frac{b}{d}}$

3

Let $\frac{5a}{6b} = \frac{6b}{7c} = \frac{7c}{8d} = m$ where $m > 0$

$\therefore 7c = 8dm$, $6b = 8dm^2$, $5a = 8dm^3$

$\therefore \sqrt[3]{\frac{5a}{8d}} = \sqrt[3]{\frac{8dm^3}{8d}} = \sqrt[3]{m^3} = m$ (1)

$, \sqrt{\frac{5a+6b}{7c+8d}} = \sqrt{\frac{8dm^3+8dm^2}{8dm+8d}}$

$= \sqrt{\frac{8dm^2(m+1)}{8d(m+1)}} = \sqrt{m^2} = m$ (2)

From (1) and (2) : $\sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a+6b}{7c+8d}}$

Lesson 4

Ex.(1): Choose

1 d	2 d	3 d	4 c
5 b	6 d	7 c	8 b
9 b	10 a	11 d	12 c
13 a	14 c	15 d	

Ex.(2): Answer the following

1	1 The variation is inverse. 2 $\because y \propto \frac{1}{x}$ $\therefore y\,x = m$ $\therefore m = 12$ 3 As $x = 3$ $\therefore 3\,y = 12$ $\therefore y = 4$ 4 As $y = 2\frac{2}{5}$ $\therefore \left(2\frac{2}{5}\right) x = 12$ $\therefore \frac{12}{5}\,x = 12$ $\therefore x = 12 \times \frac{5}{12} = 5$
2	1 $\because y \propto x$ $\therefore y = m\,x$ $\therefore 14 = 42\,m$ $\therefore m = \frac{1}{3}$ $\therefore y = \frac{1}{3}\,x$ (The relation between x , y) 2 As $x = 60$ $\therefore y = \frac{1}{3} \times 60 = 20$
3	$\because \frac{21\,x - y}{7\,x - z} = \frac{y}{z}$ $\therefore 21\,xz - zy = 7\,xy - zy$ $\therefore 21\,xz = 7\,xy$ $\therefore 3z = y$ $\therefore y \propto z$
4	$\because x^4 y^2 - 14\,x^2 y + 49 = 0$ $\therefore (x^2 y - 7)^2 = 0$ $\therefore x^2 y - 7 = 0$ $\therefore x^2 y = 7$ $\therefore y \propto \frac{1}{x^2}$
5	$\because x^2 y^2 - 6\,xy + 9 = 0$ $\therefore (xy - 3)^2 = 0$ $\therefore xy = 3$ $\therefore y \propto \frac{1}{x}$
6	1 $\because y \propto \frac{1}{x}$ $\therefore x\,y = m$ $\therefore 3 \times 2 = m$ $\therefore m = 6$ $\therefore x\,y = 6$ (The relation between x , y) 2 As $x = 1.5$ $\therefore y = \frac{6}{1.5} = 4$
7	$\because y = a - 9$, $y \propto \frac{1}{x^2}$ $\therefore y = \frac{m}{x^2}$ $\therefore \frac{m}{x^2} = a - 9$ $\therefore m = x^2 (a - 9)$ $\therefore a = 18$ as $x = \frac{2}{3}$ $\therefore m = \frac{4}{9}(18 - 9)$ $\therefore m = \frac{4}{9} \times 9 = 4$ $\therefore y = \frac{4}{x^2}$ As $x = 1$ $\therefore y = 4$
8	$\because h \propto \frac{1}{r^2}$ $\therefore \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$ $\therefore \frac{27}{h_2} = \frac{(15.75)^2}{(10.5)^2}$ $\therefore h_2 = \frac{27 \times (10.5)^2}{(15.75)^2} = 12$ cm.



Lessons 1,2

Ex.(1): Choose

1 a	2 c	3 b	4 a
5 b	6 c	7 c	8 d
9 c	10 c	11 b	12 d
13 c	14 c	15 c	

Ex.(2): Answer the following

1

1) The mean $(\bar{x}) = \frac{16+32+5+20+27}{5} = 20$

x	$x-\bar{x}$	$(x-\bar{x})^2$
16	16 − 20 = −4	16
32	32 − 20 = 12	144
5	5 − 20 = −15	225
20	20 − 20 = 0	0
27	27 − 20 = 7	49
Total		434

The standard deviation $(\sigma) = \sqrt{\frac{434}{5}} \simeq 9.3$

2) The mean $(\bar{x}) = \frac{72+53+61+70+59}{5} = 63$

x	$x-\bar{x}$	$(x-\bar{x})^2$
72	72 − 63 = 9	81
53	53 − 63 = −10	100
61	61 − 63 = −2	4
70	70 − 63 = 7	49
59	59 − 63 = −4	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.1$

2

1) The mean $(\bar{x}) = \frac{73+54+62+71+60}{5} = 64$

x	$x-\bar{x}$	$(x-\bar{x})^2$
73	73 − 64 = 9	81
54	54 − 64 = −10	100
62	62 − 64 = −2	4
71	71 − 64 = 7	49
60	60 − 64 = −4	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.07$

2) The mean $(\bar{x}) = \frac{13+14+17+19+22}{5} = 17$

x	$x-\bar{x}$	$(x-\bar{x})^2$
13	13 − 17 = −4	16
14	14 − 17 = −3	9
17	17 − 17 = 0	0
19	19 − 17 = 2	4
22	22 − 17 = 5	25
Total		54

The standard deviation $(\sigma) = \sqrt{\frac{54}{5}} = 3.286$

3

Number of children (x)	Number of families (k)	$x \times k$
0	8	0
1	16	16
2	50	100
3	20	60
4	6	24
Total	100	200

The mean of $(\bar{x}) = \frac{200}{100} = 2$ children

x	k	$x-\bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2 \times k$
0	8	0 − 2 = −2	4	32
1	16	1 − 2 = −1	1	16
2	50	2 − 2 = 0	0	0
3	20	3 − 2 = 1	1	20
4	6	4 − 2 = 2	4	24
Total	100			92

The standard deviation $(\sigma) = \sqrt{\frac{92}{100}} \simeq 1$ child

4

Number of defective units (x)	Number of boxes (k)	$x \times k$
zero	3	Zero
1	16	16
2	17	34
3	25	75
4	20	80
5	19	95
Total	100	300

The mean of $(\overline{x}) = \frac{300}{100} = 3$ units

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
Zero	3	$0 - 3 = -3$	9	27
1	16	$1 - 3 = -2$	4	64
2	17	$2 - 3 = -1$	1	17
3	25	$3 - 3 = 0$	0	0
4	20	$4 - 3 = 1$	1	20
5	19	$5 - 3 = 2$	4	76
Total	100			204

The Standard deviation $(\sigma) = \sqrt{\frac{204}{100}} \simeq 1.4$ units

5

Age (x)	Number of children (k)	$x \times k$
5	1	5
8	2	16
9	3	27
10	3	30
12	1	12
Total	10	90

The mean of $(\overline{x}) = \frac{90}{10} = 9$ years

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
5	1	$5 - 9 = -4$	16	16
8	2	$8 - 9 = -1$	1	2
9	3	$9 - 9 = 0$	0	0
10	3	$10 - 9 = 1$	1	3
12	1	$12 - 9 = 3$	9	9
Total	10			30

The standard deviation $(\sigma) = \sqrt{\frac{30}{10}} = 1.7$ years

6

Sets	centres of sets (x)	Frequency (k)	$x \times k$
0 –	2	3	6
4 –	6	4	24
8 –	10	7	70
12 –	14	2	28
16 – 20	18	9	162
Total		25	290

The mean of $(\overline{x}) = \frac{290}{25} = 11.6$

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
2	3	$2 - 11.6 = -9.6$	92.16	276.48
6	4	$6 - 11.6 = -5.6$	31.36	125.44
10	7	$10 - 11.6 = -1.6$	2.56	17.92
14	2	$14 - 11.6 = 2.4$	5.76	11.52
18	9	$18 - 11.6 = 6.4$	40.96	368.64
Total	25			800

The standard deviation $(\sigma) = \sqrt{\frac{800}{25}} \simeq 5.7$

7

Sets	centres of sets (x)	Frequency (k)	$x \times k$
5 –	6	3	18
7 –	8	6	48
9 –	10	10	100
11 –	12	12	144
13 –	14	5	70
15 – 17	16	4	64
Total		40	444

The mean of $(\overline{x}) = \frac{444}{40} = 11.1$ km./litre

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
6	3	$6 - 11.1 = -5.1$	26.01	78.03
8	6	$8 - 11.1 = -3.1$	9.61	57.66
10	10	$10 - 11.1 = -1.1$	1.21	12.1
12	12	$12 - 11.1 = 0.9$	0.81	9.72
14	5	$14 - 11.1 = 2.9$	8.41	42.05
16	4	$16 - 11.1 = 4.9$	24.01	96.04
Total	40			295.6

The standard deviation $(\sigma) = \sqrt{\frac{295.6}{40}} \simeq 2.7$ km./litre

Guide Answers

Geometry



Lesson 1

Ex.(1): Choose

1 c   2 b   3 c   4 d   5 c
6 a   7 d   8 d   9 b

Ex.(2): Answer the following

1	Let the measures of the two angles be $3x$ and $5x$ $\therefore 3x + 5x = 180°$ $\quad\therefore 8x = 180°$ $\therefore x = \frac{180°}{8} = 22.5°$ The measure of the first angle $= 3 \times 22.5° = 67.5°$ $= 67° \; 30'$ The measure of the second angle $= 5 \times 22.5°$ $= 112.5° = 112° \; 30'$
2	Let the measures of the two angles be $3x$ and $4x$ $\therefore 3x + 4x = 90°$ $\quad\therefore 7x = 90°$ $\therefore x = \frac{90°}{7} = 12\frac{6°}{7}$ $\therefore$ The measure of the greater angle $= 4 \times 12\frac{6°}{7} = 51° \; 25' \; 43''$
3	Let the measures of the interior angles of the triangle be $3x$, $4x$, $7x$ $\therefore 3x + 4x + 7x = 180°$ $\quad\therefore 14x = 180°$ $\therefore x = \frac{180°}{14} = 12\frac{6°}{7}$ The measure of the first angle $= 3 \times 12\frac{6°}{7} = 38° \; 34' \; 17''$ The measure of the second angle $= 4 \times 12\frac{6°}{7} = 51° \; 25' \; 43''$ The measure of the third angle $= 7 \times 12\frac{6°}{7} = 90°$
4	$\because m(\angle A) = 90°$ $\quad\therefore (BC)^2 = (20)^2 + (15)^2 = 625$ $\therefore BC = 25$ cm. $\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$
5	$\because m(\angle Z) = 90°$ $\therefore (ZY)^2 = (25)^2 - (7)^2$ $= 576$ $\therefore ZY = 24$ cm. 1 $\tan X \times \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$ 2 $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = \frac{625}{625} = 1$

6

$\because 2\,AB = \sqrt{3}\,AC \quad \therefore \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length unit

, $AC = 2$ length unit $\therefore BC = 1$ length unit

$\sin C = \dfrac{\sqrt{3}}{2}$, $\cos C = \dfrac{1}{2}$, $\tan C = \sqrt{3}$

7

Draw $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$, $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle , FE = 4 cm.

$\therefore$ BF + EC = 8 cm.

$\therefore$ BF = EC = 4 cm.

(Δ ABF and Δ DCE are congruent)

$\therefore$ from Δ ABF which is right-angled at F

$\therefore (AF)^2 = (5)^2 - (4)^2 = 9$

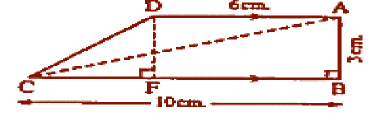
$\therefore$ AF = 3 cm. $\quad \therefore$ DE = AF = 3 cm.

(AFED is a rectangle)

$\therefore \dfrac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = \dfrac{5 \times \frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 3$

8

Draw $\overline{DF} \perp \overline{BC}$



$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$\therefore$ BF = AD = 6 cm.

$\therefore$ FC = 4 cm. , DF = AB = 3 cm.

$\therefore$ from Δ DFC which is right-angled at F

$(DC)^2 = 3^2 + 4^2 = 25 \quad \therefore$ DC = 5 cm.

$\cos(\angle DCB) - \tan(\angle ACB) = \dfrac{4}{5} - \dfrac{3}{10} = \dfrac{1}{2}$

9

$\because \dfrac{AB}{AC} = \dfrac{3}{5}$

$\therefore$ AB = 3 length unit

, AC = 5 length unit

, $\because m(\angle B) = 90°$

$\therefore (BC)^2 = 5^2 - 3^2 = 16 \quad \therefore$ BC = 4 length unit

$\therefore \sin A = \dfrac{BC}{AC} = \dfrac{4}{5}$, $\cos A = \dfrac{AB}{AC} = \dfrac{3}{5}$

, $\tan A = \dfrac{BC}{AB} = \dfrac{4}{3}$

10

Bisect $\angle$ A by the bisector $\overline{AD}$

, $\because \Delta$ ABC is an isosceles



triangle



$\therefore \overline{AD} \perp \overline{BC}$



$\therefore \sin \dfrac{A}{2} = \sin(\angle BAD)$

$\therefore \sin(\angle BAD) = \dfrac{4}{5}$



$\because \angle B$, $\angle BAD$ are acute angles



$\therefore \cos B = \sin(\angle BAD) \quad \therefore \cos B = \dfrac{4}{5}$

Lesson 2

Ex.(1): Choose

1 b	2 d	3 c	4 b
5 c	6 a	7 b	8 d
9 c	10 c	11 d	12 d
13 b	14 c	15 d	16 a

Ex.(2): Answer the following

1

1. The left side $= \sin 60° = \frac{\sqrt{3}}{2}$

   The right side $= 2 \sin 30° \cos 30°$

   $= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

   $\therefore$ The two sides are equal.

2. The left side $= \cos 60° = \frac{1}{2}$

   The right side $= 2 \cos^2 30° - 1$

   $= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

   $\therefore$ The two sides are equal.

3. The left side $= 2 \cos^2 30° - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$

   $= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

   The right side $= 1 - 2 \sin^2 30° = 1 - 2 \times \left(\frac{1}{2}\right)^2$

   $= 1 - 2 \times \frac{1}{4} = \frac{1}{2}$

   $\therefore$ The two sides are equal.

4. The left side $= \cos 60° = \frac{1}{2}$

   The right side $= \cos^2 30° - \sin^2 30°$

   $= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

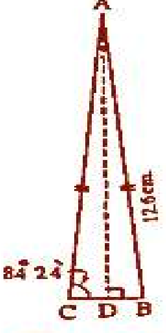
   $\therefore$ The two sides are equal.

5. The left side $= \tan 60° = \sqrt{3}$

   The right side $= \frac{2 \tan 30°}{1 - \tan^2 30°}$

   $= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$

   $\therefore$ The two sides are equal.

2	① $x \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\sqrt{3}\right)^2$ $\therefore \frac{1}{2}x = 3$ $\therefore x = 6$ ② $x \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$ $\therefore \frac{1}{4}x = \frac{3}{4}$ $\therefore x = 3$ ③ $x \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = (1)^2 - \left(\frac{1}{2}\right)^2$ $\therefore \frac{\sqrt{3}}{2}x = \frac{3}{4}$ $\therefore x = \frac{\sqrt{3}}{2}$ ④ $4x = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$ $\therefore 4x = \frac{1}{4}$ $\therefore x = \frac{1}{16}$
3	① $\because \tan x = 4 \times \frac{1}{2} \times \frac{1}{2}$ $\therefore \tan x = 1$ $\therefore x = 45°$ ② $\because \sin x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$ $\therefore \sin x = \frac{1}{2}$ $\therefore x = 30°$ ③ $\because 2\sin x = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$ $\therefore 2\sin x = 1$ $\therefore \sin x = \frac{1}{2}$ $\therefore x = 30°$
4	Draw $\overline{AD} \perp \overline{BC}$ to cut it at D $\because \overline{AD} \perp \overline{BC}$, $AB = AC$ $\therefore BD = DC$ In ΔADC : $\cos C = \frac{DC}{AC}$ $\therefore \cos 84° 24' = \frac{DC}{12.6}$ $\therefore DC = 12.6 \times \cos 84° 24' = 1.23$ $\therefore BC = 2 \times 1.23 = 2.46 \simeq 2.5$ cm.  (The req.)
5	$\because$ ABCD is a rectangle $\therefore m(\angle B) = 90°$ In ΔABC : $\therefore \sin(\angle ACB) = \frac{15}{25}$ $\therefore m(\angle ACB) \simeq 36° 52' 12''$ (First req.) In ΔABC : $\because (BC)^2 = (AC)^2 - (AB)^2$ $\therefore (BC)^2 = 625 - 225 = 400$ $\therefore BC = 20$ cm. $\therefore$ The area of the rectangle $= 15 \times 20 = 300$ cm^2. (Second req.)

6

$BC = 96 \div 8 = 12$ cm.

$\because AD = BC \qquad \therefore AD = 12$ cm. (First req.)

$\because BC = 12$ cm. , $BE = \frac{1}{4} BC \quad \therefore BE = 3$ cm.

In Δ ABE :

$\because m(\angle AEB) = 90^\circ \qquad \therefore \tan B = \frac{AE}{BE} = \frac{8}{3}$

$\therefore m(\angle B) = 69^\circ\ 26'\ 38''$ (Second req.)

In Δ AEB :

$\because \sin B = \frac{AE}{AB} \qquad \therefore \sin 69^\circ\ 26'\ 38'' = \frac{8}{AB}$

$\therefore AB = \frac{8}{\sin 69^\circ\ 26'\ 38''} \simeq 8.5$ cm. (Third req.)

Another solution :

In Δ AEB : $\because \cos B = \frac{BE}{AB}$

$\therefore \cos 69^\circ\ 26'\ 38'' = \frac{3}{AB}$

$\therefore AB = \frac{3}{\cos 69^\circ\ 26'\ 38''} \simeq 8.5$ cm. (Third req.)

A third solution :

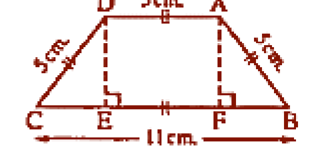
$\because \Delta$ ABE is right-angled at E

$\therefore (AB)^2 = (AE)^2 + (BE)^2$

$= 8^2 + 3^2 = 64 + 9 = 73$

$\therefore AB = \sqrt{73} \simeq 8.5$ cm. (Third req.)

7



Draw $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$, $\overline{AF} \perp \overline{BC}$,

$\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle , FE = 5 cm.

$\therefore BF + EC = 6$ cm. $\qquad \therefore BF = 3$ cm.

In Δ ABF :

$\because \cos B = \frac{BF}{AB} \qquad \therefore \cos B = \frac{3}{5}$

$\therefore m(\angle B) = 53^\circ\ 7'\ 48''$

$\therefore m(\angle A) = 180^\circ - (53^\circ\ 7'\ 48'')$

$= 126^\circ\ 52'\ 12''$ (First req.)

In Δ ABF :

$\because (AF)^2 = (AB)^2 - (BF)^2$

$\therefore (AF)^2 = (5)^2 - (3)^2 = 16 \qquad \therefore AF = 4$ cm.

$\therefore$ the area of the trapezium $= \frac{1}{2}(5 + 11) \times 4 = 32$ cm^2.

(Second req.)

Unit 5

Lessons 1,2

A

Ex.(1): Choose



A

1 d 2 c 3 b

4 a 5 b 6 c

7 c 8 d 9 c

10 d 11 a 12 c

B

1 d 2 d 3 a 4 c

5 b 6 a 7 c

Ex.(2): Answer the following

Problem [1]

$AB = \sqrt{(-1-5)^2 + (7+5)^2} = \sqrt{36+144}$

$= \sqrt{180}$ length unit

, $BC = \sqrt{(15+1)^2 + (15-7)^2} = \sqrt{256+64}$

$= \sqrt{320}$ length unit

and $CA = \sqrt{(5-15)^2 + (-5-15)^2} = \sqrt{100+400}$

$= \sqrt{500}$ length unit

, $\because (CA)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is right angled at B

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times \sqrt{180} \times \sqrt{320}$

$= 120$ square units

Problem [2]

$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

, $BC = \sqrt{(6-2)^2 + (0-3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ length units

, $AC = \sqrt{(-1-6)^2 + (-1-0)^2} = \sqrt{49+1}$

$= \sqrt{50}$ length units

, $\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at B

$\therefore$ the area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 5 \times 5$

$= 12.5$ square units.

Problem [3]

$AB = \sqrt{(0+1)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $BC = \sqrt{(5-0)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

, $CD = \sqrt{(4-5)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $DA = \sqrt{(-1-4)^2 + (1-2)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

Problem [4]

$AB = \sqrt{(5+2)^2 + (-3-4)^2} = \sqrt{49+49}$

$= \sqrt{98} = 7\sqrt{2}$ length unit.

, $BC = \sqrt{(7-5)^2 + (1+3)^2} = \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ length unit.

, $CD = \sqrt{(0-7)^2 + (8-1)^2} = \sqrt{49+49}$

$= \sqrt{98} = 7\sqrt{2}$ length unit.

, $DA = \sqrt{(0+2)^2 + (8-4)^2} = \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

Problem [5]

$\because AB = \sqrt{(0-4)^2 + (1-5)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit.

, $BC = \sqrt{(4-1)^2 + (5-8)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ length unit.

, $CD = \sqrt{(1+3)^2 + (8-4)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit.

, $AD = \sqrt{(0+3)^2 + (1-4)^2} = \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ length unit.

$\because AB = CD$, $BC = AD$

$\therefore$ the figure ABCD is a parallelogram

, $\because AC = \sqrt{(0-1)^2 + (1-8)^2} = \sqrt{1+49}$

, $\because AC = \sqrt{(0-1)^2 + (1-8)^2} = \sqrt{1+49}$

$= \sqrt{50} = 5\sqrt{2}$ length unit.

, $BD = \sqrt{(4+3)^2 + (5-4)^2} = \sqrt{49+1}$

$= \sqrt{50} = 5\sqrt{2}$ length unit.

$\therefore AC = BD = 5\sqrt{2}$ length unit.

$\therefore$ The figure ABCD is a rectangle its diagonal length $= 5\sqrt{2}$ length unit.

Problem [6]

$\because AB = \sqrt{(3-0)^2+(3-3)^2} = \sqrt{9+0} = 3$ length unit.

, $BC = \sqrt{(0-0)^2+(3-0)^2} = \sqrt{0+9} = 3$ length unit.

, $CD = \sqrt{(0-3)^2+(0-0)^2} = \sqrt{9+0} = 3$ length unit

, $DA = \sqrt{(3-3)^2+(0-3)^2} = \sqrt{0+9} = 3$ length unit.

$\because AB = BC = CD = DA$ $\quad \therefore$ ABCD is a rhombus

$\because AC = \sqrt{(3-0)^2+(3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit.

, $BD = \sqrt{(0-3)^2+(3-0)^2} = \sqrt{9+9} = \sqrt{18}$

$= 3\sqrt{2}$ length unit.

$\therefore AC = BD$

$\therefore$ The figure ABCD is a square , the length of its diagonal $= 3\sqrt{2}$ length unit ,

its area $= 3 \times 3 = 9$ square unit.

Problem [7]

$\because MA = \sqrt{(-1-3)^2+(2+1)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ length unit

, $MB = \sqrt{(-1+4)^2+(2-6)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

and $MC = \sqrt{(-1-2)^2+(2+2)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length unit

$\therefore MA = MB = MC$

$\therefore$ A , B and C lie on the circle M which its radius length is 5 length units

$\therefore$ circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.4$ length units

Problem [8]

$AB = \sqrt{(2-3)^2+(x+1)^2} = \sqrt{1+(x+1)^2}$

, $\because AB = \sqrt{17}$

$\therefore \sqrt{1+(x+1)^2} = \sqrt{17}$ "squaring the two sides"

$\therefore 1+(x+1)^2 = 17 \quad \therefore (x+1)^2 = 16$

$\therefore x+1 = \pm 4$

$\therefore x+1 = 4$ or $x+1 = -4$

$\therefore x = 3$ or $x = -5$

Problem [9]

$\because \sqrt{(a+2)^2+(7-3)^2} = 5$ "squaring the two sides"

$\therefore (a+2)^2+(4)^2 = 25 \quad \therefore a^2+4a+4+16 = 25$

$\therefore a^2+4a-5 = 0 \quad \therefore (a-1)(a+5) = 0$

$\therefore a = 1$ or $a = -5$

Problem [10]

$BC = \sqrt{(5-3)^2+(1-2)^2} = \sqrt{4+1} = \sqrt{5}$ length unit.

$\therefore AB = \sqrt{5}$ *length unit.*

$\therefore \sqrt{(x-3)^2+(3-2)^2} = \sqrt{5}$ "squaring the two sides"

$\therefore (x-3)^2+(1)^2 = 5 \quad \therefore x^2-6x+9+1 = 5$

$\therefore x^2-6x+5 = 0 \quad \therefore (x-5)(x-1) = 0$

$\therefore x = 5$ or $x = 1$

Problem [11]

Let B (x , y) $\quad \therefore (6 , -4) = \left(\frac{5+x}{2} , \frac{-3+y}{2}\right)$

$\therefore \frac{5+x}{2} = 6 \quad \therefore 5+x = 12$

$\therefore x = 7$, $\frac{-3+y}{2} = -4$

$\therefore -3+y = -8 \quad \therefore y = -5 \quad \therefore B(7 , -5)$

Problem [12]

$\because (2a-3 , a-b) = \left(\frac{7+3}{2} , \frac{-1+7}{2}\right) = (5 , 3)$

$\therefore 2a-3 = 5 \quad \therefore 2a = 8 \quad \therefore a = 4$

$a-b = 3 \quad \therefore 4-b = 3 \quad \therefore b = 1$

Problem [13]

Let : A (x , y)

$\therefore (5 , 7) = \left(\frac{x+8}{2} , \frac{y+11}{2}\right)$

$\therefore \frac{x+8}{2} = 5 \quad \therefore x+8 = 10$

$\therefore x = 2$, $\frac{y+11}{2} = 7 \quad \therefore y+11 = 14$

$\therefore y = 3 \quad \therefore A(2 , 3)$

$\therefore r = MA = \sqrt{(5-2)^2+(7-3)^2}$

$= \sqrt{9+16} = 5$ length unit.

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 5 = 31.4$ length unit

Problem [14]

$\because$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2} , \frac{2-3}{2}\right) = \left(1\frac{1}{2} , -\frac{1}{2}\right)$

$\therefore$ The point of intersection of the two diagonals is $\left(1\frac{1}{2} , -\frac{1}{2}\right)$ (Q.E.D.1)

and let D (x , y)

$\because$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore \left(1\frac{1}{2} , -\frac{1}{2}\right) = \left(\frac{x+4}{2} , \frac{y-5}{2}\right) \quad \therefore \frac{x+4}{2} = 1\frac{1}{2}$

$\therefore x+4 = 3 \quad \therefore x = -1$

, $\frac{y-5}{2} = -\frac{1}{2} \quad \therefore y-5 = -1 \quad \therefore y = 4$

$\therefore D(-1 , 4)$ (Q.E.D.2)

Problem [15]

$AB = \sqrt{(2-6)^2 + (-4-0)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit

$BC = \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length unit

$CA = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

$\because (AB)^2 + (BC)^2 = (4\sqrt{2})^2 + (6\sqrt{2})^2$

$= 32 + 72 = 104 = (CA)^2$

$\therefore \Delta ABC$ is a right-angled at B (Q.E.D.1)

Let E be the midpoint of $\overline{AC}$

$\therefore$ The coordinates of $E = \left(\frac{6-4}{2} , \frac{0+2}{2}\right) = (1 , 1)$

$\because$ In the rectangle the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let $D(x , y)$

$\therefore (1 , 1) = \left(\frac{x+2}{2} , \frac{y-4}{2}\right)$ $\therefore \frac{x+2}{2} = 1$

$\therefore x + 2 = 2$ $\therefore x = 0$

$\frac{y-4}{2} = 1$ $\therefore y - 4 = 2$

$\therefore y = 6$ $\therefore D(0 , 6)$ (Q.E.D.2)

Lessons 3

Ex.(1): Choose

1 a	2 b	3 c	4 b
5 c	6 a	7 a	8 d
9 c	10 d	11 c	12 b
13 a	14 d		

Ex.(2): Answer the following

Problem [1]

$\because m_1 = \frac{3+1}{6-2} = 1$, $m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

Problem [2]

(1) $\because m_1 = \frac{k-1}{2-3} = \frac{k-1}{-1}$, $m_2 = \tan 45° = 1$

$\because L_1 \parallel L_2$ $\quad$ $\therefore m_1 = m_2$

$\therefore \frac{k-1}{-1} = 1$ $\quad$ $\therefore k - 1 = -1$

$\therefore k =$ zero

(2) $\because m_1 = \frac{k-1}{-1}$, $m_2 = 1$

, $\because L_1 \perp L_2$ $\quad$ $\therefore m_1 \, m_2 = -1$

$\therefore \frac{k-1}{-1} \times 1 = -1$ $\quad$ $\therefore k - 1 = 1$

$\therefore k = 2$

Problem [3]

$\because m_1 = \frac{3-1}{A-0} = \frac{2}{A}$, $m_2 = \frac{5-1}{2-0} = \frac{4}{2} = 2$

$\because m_1 = m_2$ $\quad$ $\therefore \frac{2}{A} = 2$ $\quad$ $\therefore A = 1$

Problem [4]

$\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{0-3}{6-2} = \frac{-3}{4}$

, $\because m_1 \times m_2 = \frac{4}{3} \times \frac{-3}{4} = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\quad$ $\therefore \Delta ABC$ is a right-angled at B

Problem [5]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{1-3}{5+1} = -\frac{1}{3}$

, the slope of $\overleftrightarrow{CD} = \frac{6-4}{0-6} = -\frac{1}{3}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ $\quad$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \frac{6-3}{0+1} = 3$

, the slope of $\overleftrightarrow{BC} = \frac{4-1}{6-5} = 3$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ $\quad$ (2)

From (1) and (2) : we deduce that

ABCD is a parallelogram

$\because$ the slope of $\overleftrightarrow{AB} \times$ the slope of $\overleftrightarrow{BC} = -\frac{1}{3} \times 3 = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore$ The figure ABCD is a rectangle

Problem [6]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{2+2}{3-9} = -\frac{2}{3}$

$\because$ The slope of $\overleftrightarrow{AB} =$ the slope of $\overleftrightarrow{CD}$

$\therefore \frac{-X+3}{X-4} = \frac{2}{-3}$ $\quad$ $\therefore -3(-X+3) = 2(X-4)$

$\therefore 3X - 9 = 2X - 8$ $\quad$ $\therefore X = 1$

$\therefore$ The coordinates of $C = (1, -1)$

Lessons 4

Ex.(1): Choose

1 d	2 b	3 d	4 d
5 a	6 a	7 c	8 d
9 a	10 c	11 c	12 d
13 b	14 a	15 b	16 d

Ex.(2): Answer the following

Problem [1]

$y = 2x + 7$

Problem [2]

$\because$ The slope of the given straight line $= \frac{2}{3}$

$\therefore$ The slope of the required straight line $= \frac{2}{3}$

and intercepted from the negative part of y-axis 3 units

$\therefore$ The equation of the required straight line is :

$y = \frac{2}{3}x - 3$

Problem [3]

$\because$ The slope $= 2$ $\therefore y = 2x + c$

, $\because$ *the straight line passes through the point* $(2, -1)$

$\therefore -1 = 2 \times 2 + c$ $\therefore c = -5$

$\therefore y = 2x - 5$

Problem [4]

$\because$ The slope of the given straight line $= \frac{1}{2}$

$\therefore$ The slope of the required straight line $= -2$

$\therefore$ The equation of the required straight line is :

$y = -2x + c$

$\because$ The straight line passes through the point $(-2, 3)$

$\therefore 3 = -2 \times (-2) + c$ $\therefore c = -1$

$\therefore$ The equation of the required straight line is :

$y = -2x - 1$

Problem [5]

$\because$ The slope of the given straight line $= -\frac{1}{2}$

$\therefore$ The slope of the required straight line $= -\frac{1}{2}$

$\therefore$ The equation of the required straight line is

$y = -\frac{1}{2}x + c$

$\because$ The straight line passes through the point $(3, -5)$

$\therefore -5 = -\frac{1}{2} \times 3 + c$ $\therefore c = -3\frac{1}{2}$

$\therefore$ The equation of the required straight line is

$y = -\frac{1}{2}x - 3\frac{1}{2}$

Problem [6]

$\because$ The slope of the given straight line

$= \frac{2-6}{-1-5} = \frac{2}{3}$

$\therefore$ The slope of the required straight line $= \frac{2}{3}$

$\therefore$ The equation of the required straight line is

$y = \frac{2}{3}x + c$

$\because$ The straight line passes through the point $(3, 2)$

$\therefore 2 = \frac{2}{3} \times 3 + c$ $\therefore c = 0$

$\therefore$ The equation of the straight line is $y = \frac{2}{3}x$

Problem [7]

$\because$ The slope of $\overleftrightarrow{AB} = \frac{-4+3}{5-2} = -\frac{1}{3}$

$\therefore$ The slope of the required straight line $= 3$

$\therefore$ The equation of the required straight line is :

$y = 3x + c$

$\because$ The straight line passes through the point $(1, 2)$

$\therefore 2 = 3 \times 1 + c$ $\therefore c = -1$

$\therefore$ The equation of the required straight line is :

$y = 3x - 1$

Problem [8]

$\because$ The slope of the given straight line $= \tan 45° = 1$

$\therefore$ *The slope of the required straight line* $= -1$

$\therefore$ The equation of the required straight line is :

$y = -x + c$

$\because$ The straight line passes through the point $(2, -2)$

$\therefore -2 = -2 + c$ $\therefore c = 0$

$\therefore$ The equation of the required straight line is :

$y = -x$

Problem [9]

$\because$ The slope of the straight line $= \frac{1+1}{1-2} = -2$

$\therefore$ The equation of the straight line

is $y = -2x + c$

$\because$ The straight line passes through the point $(1, 1)$

$\therefore 1 = -2 \times 1 + c$ $\therefore c = 3$

$\therefore$ The equation of the straight line is

$y = -2x + 3$

Problem [10]

∵ The slope of the straight line

∴ The equation of the straight lir

∵ The straight line passes through the point (4 , 2)

∴ $2 = \frac{1}{2} \times 4 + c$ ∴ c = zero

∴ The equation of the straight line is $y = \frac{1}{2}x$

∴ The intercepted part of y-axis = zero

∴ The straight line passes through the origin point

Problem [11]

∵ The slope of the given straight line = 2

∴ The slope of the required straight line $= -\frac{1}{2}$

∴ The equation of the required straight line is $y = -\frac{1}{2}x + c$

∵ The midpoint of $\overline{AB} = \left(\frac{3-1}{2}, \frac{6+4}{2}\right) = (1, 5)$

∴ The required straight line passes through the midpoint of $\overline{AB}$

∴ $5 = -\frac{1}{2} \times 1 + c$ ∴ $c = 5\frac{1}{2}$

∴ The equation of the required straight line is $y = -\frac{1}{2}x + 5\frac{1}{2}$

Problem [12]

∵ The slope of the straight line $\overleftrightarrow{AB} = \frac{1-3}{-2-2} = \frac{1}{2}$

, The slope of the other straight line $= \frac{-1}{-2} = \frac{1}{2}$

∴ The slope of the straight line $\overleftrightarrow{AB}$ = the slope of the other straight line

∴ The two straight lines are parallel.

Problem [13]

The slope of the straight line whose equation : $2x + y + 8 = 0$ is $\frac{-2}{1} = -2$

and the slope of $\overleftrightarrow{AB} = \frac{1-3}{-2-2} = \frac{1}{2}$

, ∵ $-2 \times \frac{1}{2} = -1$

∴ The two straight lines are perpendicular.

Problem [14]

(1) At y = 0 ∴ $2x - 3 \times 0 - 6 = 0$

∴ $2x = 6$ ∴ $x = 3$

∴ The straight line cuts the x-axis at the point A (3 , 0)

, At $x = 0$ ∴ $2 \times 0 - 3y - 6 = 0$

∴ $-3y = 6$ ∴ $y = -2$

∴ The straight line cuts the y-axis at the point B (0 , −2)

(2) Let D is the midpoint of $\overline{AB}$

∴ The coordinates of $D = \left(\frac{3+0}{2}, \frac{0-2}{2}\right) = \left(\frac{3}{2}, -1\right)$

, ∵ The straight line is parallel to the y-axis

∴ Its slope is undefined

, ∵ The straight line passes through the point $D\left(\frac{3}{2}, -1\right)$

∴ The eqation of the straight line is : $x = \frac{3}{2}$

Problem [15]

∵ $m_1 = \frac{-a}{2}$, $m_2 = \frac{5-3}{1-2} = \frac{2}{-1} = -2$

, ∵ The two straight lines are parallel

∴ $m_1 = m_2$ ∴ $\frac{-a}{2} = -2$

∴ $-a = -4$ ∴ $a = 4$

Problem [16]

∵ The slope of $\overleftrightarrow{XY} = \frac{6+2}{-5-3} = -1$

∴ The slope of the axis of symmetry of $\overline{XY} = 1$

∴ The equation of the axis of symmetry of $\overline{XY}$ is : $y = x + c$

, ∵ The midpoint of $\overline{XY} = \left(\frac{3-5}{2}, \frac{-2+6}{2}\right) = (-1, 2)$

∴ (−1 , 2) satisfies the equation : $y = x + c$

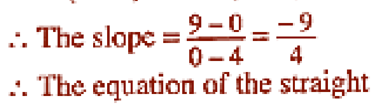
∴ $2 = -1 + c$ ∴ $c = 3$

∴ The equation of the axis of symmetry of $\overline{XY}$ is : $y = x + 3$

Problem [17]

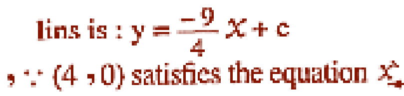
$\because$ The straight line passes trough the two points

A (4 , 0) and B (0 , 9)



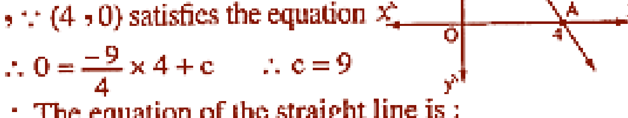
$\therefore$ The slope $= \frac{9-0}{0-4} = \frac{-9}{4}$

$\therefore$ The equation of the straight



lins is : $y = \frac{-9}{4}x + c$

, $\because$ (4 , 0) satisfies the equation



$\therefore 0 = \frac{-9}{4} \times 4 + c \qquad \therefore c = 9$

$\therefore$ The equation of the straight line is :

$y = \frac{-9}{4}x + 9$

Problem [18]

(1) $\because$ The slope of the straight line $= \frac{3-1}{2-1} = 2$

$\therefore$ The equation of the straight line is $y = 2x + c$

$\because$ The point $(1 , 1) \in$ the straight line

$\therefore 1 = 2 \times 1 + c \qquad \therefore c = -1$

$\therefore$ The equation of the straight line is $y = 2x - 1$

(2) One unit of the negative part of y-axis

(3) $\because$ The point (3 , a) satisfies the equation

$\therefore a = 2 \times 3 - 1 = 5$

Problem [19]

Let : D (x , y)

, $\because$ The midpoint of $\overline{AB}$ = The midpoint of $\overline{OD}$

, $\because$ The midpoint of $\overline{AB} = \left(\frac{6+2}{2} , \frac{6+2}{2}\right) = (4 , 4)$

, $\because$ the midpoint of $\overline{OD} = \left(\frac{0+x}{2} , \frac{0+y}{2}\right)$

$\therefore (4 , 4) = \left(\frac{x}{2} , \frac{y}{2}\right) \qquad \therefore \frac{x}{2} = 4 \qquad \therefore x = 8$

, $\frac{y}{2} = 4 \qquad \therefore y = 8$